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ABSTRACT

This text provides information and exercises on arithmetic topics which should be mastered before a student enrolls in an Elementary Algebra course. Section I describes the fundamental properties and relationships of whole numbers, focusing on basic operations, divisibility tests, exponents, order of operations, prime numbers, greatest common divisors, and least common multiples. Section II covers the addition, subtraction, multiplication, and division of integers, i.e., signed whole numbers. After Section III establishes the fundamental properties of fractions, it deals with performing basic operations with fractions and with proportions and complex fractions. Decimals are examined in Section IV, which covers decimal notation, basic operations with decimals, square roots, and percents. Section V looks at the use of integers as exponents and explains scientific notation. The last section presents rudiments of algebra, including like terms, multiplication and division of monomials, and literal formulas. In addition to interspersing reinforcement exercises throughout the explanatory text, each section concludes with a set of review problems. Concluding the text are answer keys for all of the exercises and review problems, a pretest to aid counselors and math faculty in assessing a student's basic arithmetic background, and an index. (AYC)

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PRE-ALGEBRA

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By John Kennedy
SANTA MONICA COLLEGE

TC 810 667

PREFACE

These materials have been developed under a grant from the Fund for Instructional Improvement supported by the State of California. Although my project was intended to serve the traditional instructional program in the area of Pre-Algebra, the objective was not to write a textbook. The idea was to collect in one place all those topics which I felt should be included in the Pre-Algebra course at Santa Monica College. The result is a collection of topics beyond basic arithmetic which I consider prerequisite to the study of algebra.

These materials may be used by students as well as instructors who are involved in pre-algebra courses. The topics chosen include what I think a well-prepared student should master before enrolling in an Elementary Algebra course. Not everyone will agree with my choice of topics as being both desirable and necessary. There is no agreed upon definition of what constitutes pre-algebra. My choices were based on those computational areas with which the majority of beginning algebra students have difficulty. The sole purpose is preparation for algebra.

The table of contents outlines all of the topics covered. Of significance in Section I are the order of operations and the use of grouping symbols as well as the use of prime numbers. Signed numbers are introduced in Section II, before the sections on fractions and decimals. The advantage is integrating negative numbers with fractions and decimals. Complex fractions are included in Section III and provide an important application of the distributive law. Decimal computations of square roots are avoided in Section IV. Square roots are introduced only to provide practice in reducing radicals. Section V provides background for manipulating exponents and includes scientific notation. Negative exponents as well as the standard rules for working with exponents are presented. Section VI provides practice in operating on monomials. The use of literal formulas is also included. Two topics not included are solving equations and an introduction to graphing in two dimensions.

At the end of each section are review exercises. Answers to all of the exercises are provided. A pretest is included which may aid counselors and math faculty in deciding on the adequacy of a student's basic arithmetic background for pre-algebra. Many students will be better served by taking a course in basic arithmetic before attempting pre-algebra.

Finally I would like to thank the members of the Board of Trustees as well as individuals in the administration at Santa Monica College for encouraging and supporting my efforts which have resulted in this work. A special thanks to Helen Miyashiro whose time and patience were required to type the manuscript. Any errors that remain are my sole responsibility.

Santa Monica, California J.K.

March, 1980

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BASIC OPERATIONS

The first topics studied in mathematics are numbers and their relationship to one another. The numbers that we count with

1, 2, 3, 4, 5, 6, . . .

are called the counting numbers. But these are not the only numbers that we use in mathematics or even in practical problems. Anyone who has done any cooking with a recipe or any other kind of measuring soon learns the necessity of having other numbers in our number system.

The first number we will add to our number system is 0. The whole numbers include 0 together with the counting numbers. The whole numbers are:

0, 1, 2, 3, 4, 5, 6, ...

This section is concerned with describing the fundamental properties and relationships of whole numbers.

The four basic operations used in arithmetic and algebra are addition, subtraction, multiplication, and division. If you haven't done any mathematics in a while you are about to get your chance to oil up any rusty gears and get your gray matter vibrating again.

EXERCISES: Work the following problems.

1. $\begin{array}{r} 236 \\ +114 \\ \hline \end{array}$

2. $\begin{array}{r} 958 \\ -437 \\ \hline \end{array}$

3. $\begin{array}{r} 63,149 \\ +78,697 \\ \hline \end{array}$

4. $\begin{array}{r} 2378 \\ 1359 \\ +\ 287 \\ \hline \end{array}$

5. $\begin{array}{r} 52,371 \\ -14,796 \\ \hline \end{array}$

6. $\begin{array}{r} 43,216,327 \\ -25,763,285 \\ \hline \end{array}$

7. $\begin{array}{r} 24036 \\ 31802 \\ +12791 \\ \hline \end{array}$

8. $\begin{array}{r} 31098 \\ 28736 \\ +10596 \\ \hline \end{array}$

9. $\begin{array}{r} 2634 \\ \times 8 \\ \hline \end{array}$

10. $\begin{array}{r} 487 \\ \times 345 \\ \hline \end{array}$

11. $\begin{array}{r} 385 \\ \times 39 \\ \hline \end{array}$

12. $\begin{array}{r} 4036 \\ \times 253 \\ \hline \end{array}$

13. $\begin{array}{r} 6387 \\ \times 586 \\ \hline \end{array}$

14. $\begin{array}{r} 7892 \\ \times 574 \\ \hline \end{array}$

15. A creeping charley must be watered three times a week. How many times a year (52 weeks) must it be watered?

16. If you wash dishes three times a day, how many times will you wash dishes in one year? (365 days)

17. One eight-inch stalk of celery contains five calories. How many calories are there in 197 stalks?

18. One international nautical mile equals 1852 meters. How many meters are in 2075 nautical miles?

19. For tax purposes, on fermented liquors the federal government uses a standard of 31 gallons per barrel. How many gallons are in 2078 such barrels?

20. How many gallons of water are in a swimming pool that measures 22 feet by 32 feet by 6 feet? (1 cubic foot = 7.5 gallons)

The answers to the previous exercises may be found on page 131.

We will introduce some mathematical terminology from time to time. Our purpose in doing so is to help you become mathematically literate. Every field contains some special terminology. We will give you some, but by no means all, of the jargon used in arithmetic and algebra.

The process of division can be used to illustrate applications of the other three basic operations. In a division problem the number we divide with is called the divisor. The number we divide into is called the dividend. The answer is called the quotient. If the result of the division does not come out even then we may have a non-zero number left over called the remainder. There are three different types of symbols used to indicate division. One way would be to write $318 \div 13$. But most of us would find it very difficult to give the answer when the problem is written in this form. Another form is $\frac{318}{13}$ which is a fractional form. Fractions will be covered later. The third way to indicate division is to write: $13\overline{)318}$

When we are finished our work will be arranged as:

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \\ \vdots \\ \underline{\quad\quad} \\ \text{remainder} \end{array}$$

As we work thru the problem we obtain only one digit in the quotient at a time. The first digit in the quotient should be placed above the last digit of the first number we divide into. We can't divide 13 into 318 all at once. In this problem 13 first divides into 31 two times. So the first digit of our quotient is 2 which should be written above the digit 1 in 31.

$$\begin{array}{r} 2 \\ 13\overline{)318} \end{array}$$

The process is then continued by multiplying, subtracting, and bringing down the next digit.

$$\begin{array}{r} 2 \\ 13\overline{)318} \\ \underline{26} \\ 58 \end{array}$$

The next digit in the quotient is 4 and when we finish we have:

$$\begin{array}{r} 24 \\ 13\overline{)318} \\ \underline{26} \\ 58 \\ \underline{52} \\ 6 \end{array}$$

Answer: quotient = 24 remainder = 6

Each number we divide into must be at least as large as the divisor. The result of each subtraction should be a number smaller than the divisor. We can check our work by performing the following operations.

"Division Check"

$$\text{quotient} \times \text{divisor} + \text{remainder} \stackrel{?}{=} \text{dividend}$$

The check is whether or not our answer equals the dividend. You should know this division check because we will use a similar principle when we convert between improper fractions and mixed numbers. Study the next sample problem before doing the next set of exercises. Why does the digit 0 appear in the quotient?

Sample:

$$\begin{array}{r} 307 \\ 17\overline{)5232} \\ \underline{51} \\ 132 \\ \underline{119} \\ 13 \end{array}$$

Answer: quotient = 307

remainder = 13

EXERCISES: Perform the following divisions giving your answers as quotients and remainders.

1. $37\overline{)325}$
2. $25\overline{)880}$
3. $17\overline{)673}$
4. $64\overline{)321}$
5. $70\overline{)983}$
6. $97\overline{)2528}$
7. $27\overline{)7709}$
8. $19\overline{)2637}$
9. $83\overline{)1377}$
10. $145\overline{)4878}$
11. $32\overline{)1091}$
12. $407\overline{)4065}$

13. $269\overline{)18532}$

14. $316\overline{)19764}$

15. $686\overline{)40344}$

16. $560\overline{)2725}$

17. $61\overline{)305122}$

18. $942\overline{)85660}$

19. $39\overline{)312160}$

20. $256\overline{)2292}$

The answers to the previous exercises may be found on page 131.

DIVISIBILITY TESTS

Now that we have had an opportunity to begin working some math problems we will look at some special properties of numbers. Eventually we will be concerned with prime numbers, but first we need to develop a few special techniques. What follows are four special tests which tell when certain division problems come out even (have zero remainder).

1. A whole number is evenly divisible by 2 if the number is an even number which means its last digit is 0,2,4,6, or 8.

2. A whole number is evenly divisible by 3 if the sum of its digits is evenly divisible by 3.

3. A whole number is evenly divisible by 5 if its last digit is 0 or 5.

4. A whole number is evenly divisible by 10 if its last digit is 0.

2 divides evenly into 4776 because the last digit in 4776 is 6. 2 does not divide evenly into 319 because the last digit is 9, 319 is an odd number.

Does 3 divide evenly into 5637? To test this we add the digits of 5637 together. 5+6+3+7 = 11+3+7 = 14+7 = 21. Since 3 divides evenly into 21, 3 will also divide evenly into 5637. We can even apply the same test to the number 21. 2+1 = 3 and 3 divides evenly into 3.

On the other hand, 3 does not divide evenly into 4736 because when we sum the digits we have 4+7+3+6 = 20 and 3 does not divide evenly into 20.

Try not to confuse the divisibility test for 3 with the other divisibility tests. In the previous example we saw that 3 did not divide evenly into 4736. The last digit in 4736 is 6 and 3 does divide evenly into 6, but the divisibility test for 3 has nothing to do with whether or not 3 divides evenly into the last digit. To test divisibility by 3, sum the individual digits of the number. Do <u>not</u> sum the digits when testing for division by 2 or 5 or 10.

4736 is not evenly divisible by 5 because the last digit is neither a 5 or a 0. 10 does divide evenly into 1580 because the last digit is a zero.

None of these divisibility tests tell us what the answer is to the division problem. These tests are simply shortcuts that tell us whether or not our answers should come out even. When we study prime numbers later on we will be asking ourselves if 2 or 3 or 5 divide evenly into certain numbers. Knowing these shortcuts will save us work. There is no reason to perform a long division problem when we don't have to.

EXERCISES: There are four answers to each problem below. Tell whether or not 2, 3, 5, and 10 divide evenly into each number. It

isn't necessary to actually perform the divisions unless you want to check your answers.

Sample: 5984

Answers: 2 yes 3 no 5 no 10 no

1.	3695	6.	35,560	11.	4672	16.	73025
2.	120	7.	243	12.	221	17.	140,725
3.	2,510	8.	743	13.	6000	18.	170,600
4.	6,714	9.	13,145	14.	1532	19.	252,689,241
5.	8,280	10.	31,955	15.	3,337	20.	9,681,849,520

The answers to the previous exercises may be found on page 131.

When we first learned arithmetic we were taught how to add before we learned how to multiply. When we multiply 5x6 we can think of repeated addition where the first number is the number to be added to itself and the second number tells us how many times to write the first number. Thus, 5x6 = 5 + 5 + 5 + 5 + 5 + 5. We have written six 5's to be added. This example shows that multiplication is equivalent to repeated addition. The addition operation is being repeated.

There is another notation used in mathematics which indicates repeated multiplication. This is written as follows:

$$5x5x5x5x5x5 = 5^6$$

Just as before, the number 6 is telling us how many 5's are written. The difference here is that the 5's are now multiplied together and the number 6 is not written on the same line, 6 is written as a superscript just above the 5.

Another example is:

$$6x6x6x6x6 = 6^5$$

Here the number 6 is repeated five times. Note that the number of times 6 is repeated is written as a superscript above 6. The order here is important.

In ordinary multiplication it doesn't make any difference if we write 3x4 or 4x3. The answer is always 12. But there is a significant difference between 4^3 and 3^4.

$$4^3 = 4x4x4 = 64 \text{ and } 3^4 = 3x3x3x3 = 81$$

The bottom number is called the <u>base</u> and the top number is called the <u>exponent</u>. This form of notation is called exponential notation. The exponent tells you how many times to repeat the base. The operation is repeated multiplication. You never multiply the base times the exponent. Thus, $4^3 \neq 12$ and $3^5 \neq 15$

"5^6" is read as "5 to the 6th power" or "5 to the power of 6"

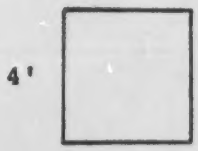
"3^4" is read as "3 to the 4th power" or "3 to the power of 4"

"4^3" is read as "4 to the 3rd power" or "4 to the power of 3"

There are special names associated with 2nd and 3rd powers. These names are related to area and volume. The formula for the area of a rectangle is:

area = length x width

Now if our rectangle is a square which measures 4 feet on a side (as pictured below)



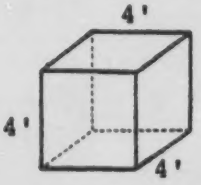
then length = width and the area = $4x4 = 4^2 = 16$ square feet.

2^{nd} powers are also called squares. "4^2" may be read as "4 squared."

The formula for the volume of a cube is:

volume = length x width x height

If our cube measures 4 feet on a side as pictured below



then length = width = height and

volume = $4x4x4 = 4^3 = 64$ cubic feet.

3^{rd} powers are also called cubes. 4^3 may be read as "4 cubed."

Sometimes people say 4^3 means multiply 4 by itself three times. This is not literally true, for when we write 4^3 using repeated multiplication we write $4^3 = 4x4x4$ which shows the base number 4 written three times, but note that there are only two multiplications. The number of multiplications is always one less than the exponent.

It is also possible to use 0 and 1 as exponents. When we write 7^1 the exponent 1 tells us how many 7's to write, just one. There are no multiplications to be performed so $7^1 = 7$. Any number to the first power equals itself. We will define 0 exponents when we develop some rules for working with exponents.

EXERCISES: Compute the following powers. Write your answers as single numbers.

1. $2^3 =$	6. $2^6 =$	11. $1^{1000} =$	16. $2^3x2^2x2^1 =$
2. $3^2 =$	7. $6^2 =$	12. $0^3 =$	17. $2^4x5^2 =$
3. $7^3 =$	8. $0^3 =$	13. $2^2x3^2 =$	18. $2^3x5^1x7^2 =$
4. $16^2 =$	9. $1^4 =$	14. $3^4 =$	19. $3^2x5^3x11^1 =$
5. $5^3 =$	10. $0^{100} =$	15. $3^1x3^3 =$	20. $129^1 =$

21. $2^2x3^1x5^2 =$	26. $2^2x3^2x5^2 =$
22. $2^4x3^2x5^2 =$	27. $3^2x3^3x3^1 =$
23. $2^1x3^3x7^2 =$	28. $2^1x3^1x5^2x7^1 =$
24. $3^2x5^2x7^2 =$	29. $2^2x3^2x7^2x13^1 =$
25. $2^3x5^1x11^2 =$	30. $2^3x3^2x5^1x7^2 =$

The answers to the previous exercises may be found on page 132.

The following is a curious little problem. How much is 2 + 3 x 7? What we are interested in is how we form the answer to the problem. The numbers are small and the computations are about as simple as we could make them. Some people think the answer is 35 and others think the correct answer is 23. How do you get 35? Well, adding 2 and 3 gives 5 and 5 times 7 is 35. How do you get 23? First multiply 3 times 7 and then add 2.

The answer you get depends on which operation you perform first. If you add first then you will get 35 as an answer. If you multiply first you get 23. It would appear then that the problem has two answers.

In order to avoid these double answers, mathematicians have adopted certain conventions about the order in which operations are to be performed. A convention is simply an agreed upon method which acts as a standard. If you want your answers to agree with everyone else then you'll have to remember My Dear Aunt Sally. The order of the letters MDAS tells you the order in which to multiply, divide, add, or subtract. Since M comes before A we should multiply before we add if we want to be in step with the rest of the world. If we adopt this convention then 2 + 3 x 7 = 23. 23 is the correct answer.

Now it may happen that we really need to work a similar problem in another way. Consider the problem in which we want to subtract 3 from 8 and then multiply that answer by 6. Writing 8 - 3 x 6 won't do because written this way, the multiplication should be performed before the subtraction. M comes before S. The order in which we write the problem is not related to the order in which computations are performed mentally. Mathematics allows us to get around this by inserting parentheses.

The meaning of (8-3) x 6 is subtract first and then multiply. Parentheses act as grouping symbols and allow us to force the order of operations the way we want. In the problem (8-3) x 6 we can't multiply by 6 until we know the number to multiply with. The only way to figure that number is to perform the operation within parentheses first.

Sometimes there is more than one set of parentheses. The rule in this case is to start with the inner-most set of parentheses and then work outwards. My Dear Aunt Sally applies within each set of parentheses.

Example:
```
   9 x (8+(5x4-3) x 6)
= 9 x (8+(20-3) x 6)
= 9 x (8+17x6)
= 9 x (8+102)
= 9 x 110
= 990
```

Another curious little problem that Aunt Sally can't help us with is 16 ÷ 8 ÷ 2. The order in which the division operations are performed again gives rise to two different answers. 16 ÷ 8 = 2 and 2 ÷ 2 = 1.

On the other hand, $8 \div 2 = 4$ and $16 \div 4 = 4$. Another convention we apply is that similar operations should be carried out in left to right order as they are read. But in order to satisfy the MDSA convention, we do all multiplications and divisions first, then we do any additions or subtractions. So $16 \div 8 \div 2 = 2 \div 2 = 1$. 1 is the correct answer.

Examples:
$$\begin{aligned} & (35+7) \div (18-12) \times 4 \\ = \ & (42) \div (6) \times 4 \\ = \ & 7 \times 4 \\ = \ & 28 \end{aligned}$$

$$\begin{aligned} & 5 \times ((37-7x2) - (19-3x4) \times 3) \\ = \ & 5 \times ((37-14) - (19-12) \times 3) \\ = \ & 5 \times (23-7x3) \\ = \ & 5 \times (23-21) \\ = \ & 5 \times 2 \\ = \ & 10 \end{aligned}$$

Powers also come into play when we talk about the order of operations. Powers take precedence over the 4 basic operations. Also, an exponent applies to the least possible part of an expression.

This means that when we write 3×5^2 the power 2 applies only to the 5, not the 3. We perform the power before multiplying.

$$3 \times 5^2 = 3 \times 25 = 75$$

If we want the exponent 2 to apply to both the 3 and the 5 then we should use parentheses and write $(3x5)^2 = (15)^2 = 225$. We still perform the operations inside parentheses first.

Example:
$$\begin{aligned} & 3 \times (2+5x3^2)^2 \\ = \ & 3 \times (2+5x9)^2 \\ = \ & 3 \times (2+45)^2 \\ = \ & 3 \times (47)^2 \\ = \ & 3 \times 2209 \\ = \ & 6627 \end{aligned}$$

The following is a better description of the true order of operations than My Dear Aunt Sally. The letters MDAS can be misleading because they could just as well be written as DMSA. Division and multiplication come before subtraction and addition. There are times when we divide before we multiply and there are also times when we subtract before we add.

Example:
$$\begin{aligned} & 50 - 24 + 18 \div 3 \times 2 \\ = \ & 50 - 24 + 6 \times 2 \\ = \ & 50 - 24 + 12 \\ = \ & 26 + 12 \\ = \ & 38 \end{aligned}$$

We summarize the order of operations as follows-- within each set of parentheses:

1) First perform any powers.
2) Next do any multiplications and divisions.

3) Lastly do any additions or subtractions.

Each of the above 3 steps should be carried out in left to right order within groups of parentheses.

EXERCISES: Compute the following. Give your answers as single numbers.

1. $18 - 2 \times 7$
2. $4 \times 9 - 3$
3. $2 + 5 \times 6^2$
4. $7 \times (19-4) \times 2$
5. $(18-7) \times (6+2)$
6. $5^2 + 3^2$
7. $(5+3)^2$
8. $5^2 + 2 \times 5 \times 3 + 3^2$
9. $1540 - 7 \times (2^2+8)^2$
10. $(16\times8+3)^2 - (9\times3-4)^2$
11. $(16\times8+3^2) - (9\times3-4^2)$
12. $(23\times0+14) \times (13-7) \div (3^2-5)$
13. $(19-9\times2)^{34} \times (48-3\times4^2)^{16}$
14. $((6\times3+5)\times3+2) \times 3 - 8$
15. $(((5\times2-4)\times2+6)\times2+7) \times 3 + 8$
16. $(((6\times7-4)\times7+3)\times7-5) \times 7 - 13$
17. $(((8\times2^2+3)\times2^2-9)\times2^2-7) \times 2^2 + 13$
18. $7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 4$
19. $6 \times 10^4 + 2 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 5$
20. $(7+4)^2 - (7+3)^2$
21. $4^2 - (6-(20-18))$
22. $12 - (10-(8-(6-(4-2))))$
23. $2 \times (8-2\times(6-2\times(5-3)))$
24. $3 \times (7+3\times(8+3\times(16-3\times(3\times5-12))))$
25. $2^2 \times (9+2^2\times(17+2^2\times(31+2^2\times(3^3-4))))$

The answers to the previous exercises may be found on page 132.

PRIME NUMBERS:

The prime numbers are the building blocks from which all whole numbers may be constructed by using the single operation of multiplication. Prime numbers are important because they are used to reduce fractions and find least common multiples and greatest common divisors. We will also use these numbers to simplify square roots. Prime numbers are used in many areas of mathematics and computer science.

Definition: A prime number is any whole number greater than 1 which is evenly divisible only by itself and by the number 1.

For example, the number 7 is prime. The only two numbers which divide evenly into 7 are 1 and 7. The number 6 however, is not prime since 6 is evenly divisible by 2 and 3. The word "only" in the above definition of a prime number is important in understanding the prime number concept. Note that the number 6 is also evenly divisible by 1 and 6. But this is a property that is true of every number. Every number is evenly divisible by itself and one, but the prime numbers have the additional property that only the number one and the prime itself divide into the prime.

We say 2 is a divisor of 6 because 2 divides evenly into 6. The number 6 has four divisors; they are 1,2,3, and 6. The number 7 has exactly two divisors, namely 1 and 7. Every prime number has two and only two divisors. Numbers with more than two divisors are not prime and are called composite numbers. So when we are asked the question, "Is this number a prime number?" We should try to think up at least three numbers which will divide evenly into the given number.

Is 25 a prime number? Right away we can think of two divisors, 1 and 25. Is there a third number which divides evenly into 25? Yes! 5. The three divisors of 25 are 1, 5, and 25. 25 is not prime. 25 is composite.

Now let's list a few numbers.

1 has only one divisor, namely 1. Therefore 1 is not prime because it doesn't have exactly two divisors. 1 is not composite either because it doesn't have 3 or more divisors. 1 is neither prime nor composite.

The divisors of 2 are: 1, 2	2 is prime
The divisors of 3 are: 1, 3	3 is prime
The divisors of 4 are: 1, 2, 4	4 is composite
The divisors of 5 are: 1, 5	5 is prime
The divisors of 6 are: 1, 2, 3, 6	6 is composite
The divisors of 7 are: 1, 7	7 is prime
The divisors of 8 are: 1, 2, 4, 8	8 is composite
The divisors of 9 are: 1, 3, 9	9 is composite
The divisors of 10 are: 1, 2, 5, 10	10 is composite
The divisors of 11 are: 1, 11	11 is prime

The divisors of 12 are:	1, 2, 3, 4, 6, 12	12 is composite
The divisors of 13 are:	1, 13	13 is prime
The divisors of 14 are:	1, 2, 7, 14	14 is composite

The most fundamental and most useful property of prime numbers can be illustrated by breaking down the number 60 into a product of other numbers.

$$60 = 2 \times 2 \times 3 \times 5$$

We have written the number 60 as a product using the numbers 2, 3, and 5 (we used the number 2 twice). What is important however, is that 2, 3 and 5 are all prime numbers. We can do this for any number!

> Any large whole number can be written as a product of prime numbers.

This last statement is one of the most fundamental properties of our number system. It is an indication of why the prime numbers are the key elements in our system.

You may find the following three problems difficult to work at this point, but once you have mastered prime numbers, you will be able to work these problems very easily. You need only read these problems as examples now, and you can come back and work them after you have learned how to find all the prime divisors of a number.

1. What is the smallest number that both 48 and 56 divide into evenly?
2. What is the largest number that divides into both 1680 and and 1080?
3. What does the fraction $\frac{1547}{3757}$ become when written in reduced form?

To answer the above three questions we first need to find all the prime divisors of the given numbers. The following is a list of the first few prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

This list contains the primes that we need to solve most practical problems, but you should be aware that the list of prime numbers is endless. The number of primes is infinite.

To find all the prime numbers that divide into a given number we begin with the smallest prime and try to work our way thru the list of primes. Only after having tried all the previous prime numbers do we move up to the next prime in our list.

Example: Write 1,050 as a product of prime numbers.

The first prime in the list is 2. 2 divides evenly into 1050 with a quotient of 525. We can turn our answer of the division into a statement about multiplication by writing:

$$1{,}050 = 2 \times 525$$

2 is prime but 525 is composite so we have more work to do before we are finished writing 1,050 as a product of prime numbers. 3 is the next prime, and when we try to divide 3 into 525 the quotient is 175. Continuing to write everything in terms of multiplication we now have:

$$1{,}050 = 2 \times 3 \times 175$$

We might be finished except for the fact that 175 is composite. So we try the next prime 5 and we find:

$$1{,}050 = 2 \times 3 \times 5 \times 35$$

Finally, breaking 35 down as 5 x 7 gives us: $1{,}050 = 2 \times 3 \times 5 \times 5 \times 7$

There is another way to arrange our work which makes the division process go faster. We will draw a series of brackets to enclose our quotients and we always write the primes we are dividing with to the left of each bracket. The previous problem would start out as:

$$2\overline{)1050}$$

When we divide 2 into 1050 the quotient is 525. We show this as:

$$\begin{array}{r} \\ 2\overline{)1050} \\ \quad 525 \end{array}$$

Now we box the number 525 and start with the next prime 3.

$$\begin{array}{r} \\ 2\overline{)1050} \\ 3\overline{)525} \end{array}$$

The next quotient is 175:

$$\begin{array}{r} \\ 2\overline{)1050} \\ 3\overline{)525} \\ \quad \boxed{175} \end{array}$$

Continuing we have:

$$\begin{array}{r} \\ 2\overline{)1050} \\ 3\overline{)525} \\ 5\overline{)175} \\ \quad 5\overline{)35} \end{array}$$

and finally:

$$\begin{array}{r} \\ 2\overline{)1050} \\ 3\overline{)525} \\ 5\overline{)175} \\ \quad 5\overline{)35} \\ \quad\quad 7 \end{array}$$

We know we are finished at this point because our last quotient 7 is prime. Notice how the primes outside the brackets form an increasing sequence, progressing from 2 thru 7. It is customary to use exponents when writing the final product. Since the 5 above occurs twice we have as our final answer:

$$1050 = 2 \times 3 \times 5^2 \times 7$$

Example: Write 200 as a product of powers of prime numbers

$$\begin{array}{r|l} 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ & 5 \end{array} \qquad 200 = 2^3 \times 5^2$$

Some helpful hints when finding the prime factors of a number are:

1) Always begin with the smallest prime.

2) Keep repeating the same prime until that prime no longer divides evenly before trying the next prime. Use your divisibility tests for 2, 3, and 5.

3) The last number is prime when the trial quotient is smaller than the prime divisor. Keep going down the list of primes until the prime divisor is larger than the trial quotient.

The first two suggestions were followed in the example using 200. We start with 2 and after the first division we have:

$$\begin{array}{r|l} 2 & 200 \\ & 100 \end{array}$$

At this point we may recognize that 5 will divide evenly into 100 and it is tempting to next write:

$$\begin{array}{r|l} 2 & 200 \\ 5 & 100 \\ & 20 \end{array}$$

This violates the second suggestion. Keeping the primes in order really does make the computations simpler and when you are finished it's very simple to recognize what the exponents should be.

The third suggestion can be illustrated as:

Example: Is 221 prime?

If 221 were composite we could find at least two primes smaller than 221 whose product would equal 221. Beginning with the prime 2 and going right down the list of primes we find none of 2,3,5,7,11 divide evenly into 221. When dividing 11 into 221 the trial quotient is 20 with a remainder of 1. Since 20 is larger than 11 we keep going. Trying 13 we immediately find

$$\begin{array}{r|l} 13 & 221 \\ & 17 \end{array}$$

So 221 = 13 x 17 and 221 is not prime, it is composite.

Example: Is 223 prime?

As in the previous example we find that none of 2,3,5,7,11 divide evenly into 223. So we try to divide by 13. The

answer doesn't come out even, but what is important is that the trial quotient is 17 with a remainder of 2. Because 17 is larger than the prime divisor 13 we must keep going. The next prime divisor to try is 17. Dividing 17 into 223 yields a quotient of 13 and a remainder of 2. This time the prime divisor 17 is larger than the trial quotient. Choosing larger trial divisors will only produce smaller trial quotients and since we have checked all the primes up thru 17 we need not go any further. 223 is prime.

EXERCISES: Find all the prime divisors of each of the following numbers. Write your answers showing each number equal to a product of powers of its prime divisors.

Sample: 4500

$$\begin{array}{r|l} 2 & 4500 \\ 2 & 2250 \\ 3 & 1125 \\ 3 & 375 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

Answer: $4500 = 2^2 \times 3^2 \times 5^3$

1. 210	2. 120	3. 900	4. 15,400	5. 315
6. 2695	7. 1001	8. 286	9. 440	10. 442
11. 108	12. 500	13. 4851	14. 437	15. 93
16. 53	17. 229	18. 178	19. 159	20. 34,300
21. 13,750	22. 5,915	23. 21,413	24. 493	25. 961

The answers to the previous exercises may be found on page 133.

GREATEST COMMON DIVISORS AND LEAST COMMON MULTIPLES

Let's apply what we have learned about prime numbers to solve more practical problems. In this section we will work on problems dealing with least common multiples and greatest common divisors. We need to know how to find least common multiples and greatest common divisors to be able to manipulate fractions.

The least common multiple of two numbers is the smallest number that the two numbers will divide into evenly. For example, the least common multiple of 6 and 8 is 24. Both 6 and 8 divide evenly into 24, and 24 is the smallest number divisible by both 6 and 8. Of course, there are many numbers that both 6 and 8 divide into evenly; 48, 72, 96, 120, . . . 24 is the smallest such number.

Although we see the word "least" when we read least common multiple, in general the least common multiple of two numbers is larger than either number. Least common multiple is also abbreviated LCM. The LCM of 6 and 8 was larger than both 6 and 8. As another example, the least common multiple of 12 and 15 is 60. 12 divides evenly into 60 five times, and 15 divides evenly into 60 four times. Note that 60 is larger than both 12 and 15, but we call 60 their least common multiple. Least common multiples are used in adding fractions.

It may happen that the LCM of two numbers is the larger of the two numbers. This would be true if our numbers were 9 and 36. The LCM of 9 and 36 is 36. The reason is that 9 already divides evenly into 36, and since 36 divides evenly into itself, 36 is the smallest (least) number that both 9 and 36 divide into evenly. The LCM of two numbers is at least as big as the larger of the two numbers and for most pairs of numbers the LCM is larger than either number.

Sometimes the easiest way to find the LCM of two numbers is to just guess. But when the numbers are large it is very difficult to make a correct guess. The following procedure using prime numbers eliminates all the guess work.

Example: What is the least common multiple of 2100 and 720?

First find the prime divisors of both numbers and write each number as a product of powers of its prime divisors.

2 | 2100	2 | 720
2 | 1050	2 | 360
3 | 525	2 | 180
5 | 175	2 | 90
5 | 35	3 | 45
7	3 | 15
	5

$2100 = 2^2 \times 3^1 \times 5^2 \times 7^1$ $\qquad$ $720 = 2^4 \times 3^2 \times 5^1$

Next, temporarily ignoring the exponents, write down the primes which occur as divisors.

2 3 5 7

In this example 2, 3, and 5 were common to both numbers and 7

occured in 2100 but not in 720. When finding the LCM, write down all the primes which occur in either number. Don't leave out a prime just because it came up in only one of the two numbers. Always write down as many distinct primes as possible, but write each prime only once.

The final step is to fill in the exponents. The rule here is to choose the larger exponent, if there's a choice. Here's what we mean. Since the prime 2 occured in both numbers, we must choose between 2^2 and 2^4. We should choose the larger exponent 4. The prime 3 also occurred in both numbers, once as 3^1 and the other time as 3^2. We should choose the exponent for the 3 to be 2. The choice of exponent for 5 is also 2. Now since 7 occurred only in the first number, there is no choice to be made in choosing the exponent on 7. We must take 7^1. After choosing the exponents we multiply out the primes with their exponents to obtain our answer as a single number.

$$\begin{aligned} \text{LCM} &= 2^4 \times 3^2 \times 5^2 \times 7^1 \\ &= 16 \times 9 \times 25 \times 7 \\ &= 144 \times 175 \\ &= 25{,}200 \end{aligned}$$

The LCM of 2,100 and 720 is 25,200.

We summarize the steps to find the LCM of two numbers as:

1) Write each number as a product of powers of primes.
2) Write down without exponents all the primes which occur in either number.
3) Choose new exponents. If the same prime occurs in both numbers, choose the larger exponent. Otherwise, if a prime occurs in only one of the numbers, take the given exponent.
4) Multiply out the powers of the primes to obtain the answer as a single number.

EXERCISES: Find the least common multiple (LCM) of the following pairs of numbers. Write your answers in exponential form and as single numbers.

Sample: What is the LCM of 1,960 and 1,100?

2	1960	2	1100
2	980	2	550
2	490	5	275
5	245	5	55
7	49		11
	7		

$1960 = 2^3 \times 5^1 \times 7^2$ $\qquad$ $1100 = 2^2 \times 5^2 \times 11^1$

The primes are: 2 5 7 11

Answer: $\text{LCM} = 2^3 \times 5^2 \times 7^2 \times 11^1 = 107{,}800$

1. 48 and 108 $\qquad$ 2. 28 and 36 $\qquad$ 3. 42 and 56 $\qquad$ 4. 18 and 64

5. 12 and 52 6. 24 and 40 7. 54 and 30 8. 46 and 20

9. 13 and 62 10. 360 and 1500

The answers to the previous exercises may be found on page 133.

The greatest common divisor of two numbers is the largest number that divides evenly into both numbers. For example, the greatest common divisor of 30 and 42 is 6. 6 divides into both 30 and 42. Although we see the word "greatest" when we read greatest common divisor, in general the greatest common divisor of two numbers is smaller than both numbers. Greatest common divisor is abbreviated as GCD.

It may happen that the greatest common divisor of two numbers is the smaller of the two numbers, or the GCD may simply be the number 1. For example, the GCD of 8 and 24 is 8, but the GCD of 7 and 24 is 1. Greatest common divisors are used to reduce fractions. As with LCM's, finding the GCD of two numbers can sometimes be accomplished by simple guessing. But prime numbers can be used where guessing methods fail.

To find the GCD of two numbers:

1) Write each number as a product of powers of primes.
2) Write down without exponents only those primes which are common to both numbers.
3) New exponents are obtained by choosing the smaller exponent from each prime.
4) Multiply out the powers of the primes to obtain the answer as a single number.

Now go back and compare these four steps for finding the GCD with the steps for finding the LCM. The real differences occur in steps 2) and 3). You must know both procedures and be able to distinguish between them. Don't confuse LCM's with GCD's.

EXERCISES: Find the greatest common divisor (GCD) of the following pairs of numbers. Write your answers in exponential form and as single numbers.

Sample: What is the GCD of 1,680 and 4,900?

2	1680	2	4900
2	840	2	2450
2	420	5	1225
2	210	5	245
3	105	7	49
5	35		7
	7		

$1{,}680 = 2^4 \times 3^1 \times 5^1 \times 7^1$ $4{,}900 = 2^2 \times 5^2 \times 7^2$

The common primes are: 2 5 7

Answer: GCD $= 2^2 \times 5^1 \times 7^1 = 140$

1. 1500 and 360 2. 150 and 504 3. 525 and 693

4. 2002 and 2805 5. 2700 and 2250 6. 400 and 560

7. 104 and 338 8. 525 and 2205 9. 6125 and 595

10. 4704 and 756

The answers to the previous exercises may be found on page 133.

FUNDAMENTAL PROPERTIES

We have now covered most of the basics of the arithmetic of whole numbers. There are still a few laws dealing with numbers that are most useful in algebra. An understanding of these laws based on numerical examples will be of great benefit when you see them again using the language of algebra.

The first law tells us about the importance of the order in which we write numbers in operations. Note that $3 + 5 = 5 + 3$. The order in which we write the numbers in an addition problem doesn't make any difference. This is true for all numbers, not just 3 and 5. But in division, order is very important. $8 \div 4 \neq 4 \div 8$. Multiplication is like addition; order does not really matter in multiplication. $4 \times 6 = 6 \times 4$ and this property holds for any two numbers. On the other hand, order is important in subtraction, $8 - 5 \neq 5 - 8$. In the next section we will learn how to compute $5 - 8$, but for now, such a problem doesn't make any sense.

The single operations of addition and multiplication are said to be commutative because the written order doesn't make a difference in the value of the answer. The commutative law holds for addition and multiplication. Both division and subtraction violate the commutative law. In division and subtraction the written order of the numbers does make a difference in the answer.

The commutative law is concerned only with the order in which we write a problem. The next law deals with the order in which we perform operations. Think of what happens when you put on a pair of pants. As long as you don't put them on backwards, it doesn't make any difference if you start with your left leg or your right leg. But when it comes to putting on your shoes and socks there is a real difference between putting on your socks first and your shoes second or vice versa. If you don't think so, try it the other way around.

The associative law requires two operations to be performed to test the law. $5 + 8 + 7$ contains two addition operations. It doesn't make any difference if we first add 5 and 8 and then add 7, or, if we first add 8 and 7 and then add the 5. We indicate this by using parentheses to indicate the two different orders in which the operations can be performed.

$$(5+8) + 7 = 5 + (8+7)$$

This property of addition works for any three numbers. The same property holds for multiplication.

$$(4 \times 3) \times 9 = 4 \times (3 \times 9)$$

Both addition and multiplication are associative.

Neither subtraction nor division are associative. Compute the following to understand why.

$$(32 \div 8) \div 4 = ? \qquad 32 \div (8 \div 4) = ?$$

$$(27-13) - 5 = ? \qquad 27 - (13-5) = ?$$

The basic difference between the commutative and associative laws is that the commutative law deals with the physical order in which we

write a problem whereas the associative law deals with the mental order in which we perform the same operation twice. Note that the commutative law always involves a different left to right order of the numbers (they are always turned around) whereas the associative law keeps the numbers in the same left to right order but always includes parentheses in two different ways.

ADDITION MULTIPLICATION	Commutative and Associative laws are valid
SUBTRACTION DIVISION	Commutative and Associative laws are <u>not</u> valid

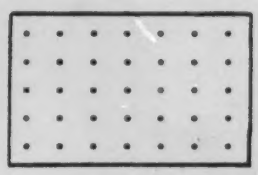
The last fundamental property that we will discuss is perhaps the most important law in all of algebra. The <u>distributive law</u> shows a strong connection between multiplication and addition. What follows is an example.

$$5 \times (4+3) = 5x4 + 5x3$$

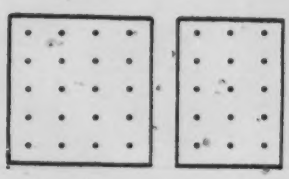
This example shows how we can compute the same problem in two different ways.

$$5 \times 7 = 35 \text{ and } 20 + 15 = 35$$

To help you gain an understanding of this property, count the number of dots in the box below.



If you are clever you didn't count each individual dot. You probably counted the number of dots across and then counted down one side and simply multiplied to get the total number of dots. The next picture shows the same number of dots split up into two different boxes.



To find the total now you can simply find the number of dots in each box as before and then add your two answers. The distributive law tells you that you will get the same count for the total number of dots regardless of how you box them.

$$\begin{aligned} 5 \times (4+3) &= 5x4 + 5x3 \\ 5x7 &= 20 + 15 \\ 35 &= 35 \end{aligned}$$

5 x (4+3) is a product which involves a sum. 5x4 + 5x3 is a sum of two products. Notice how the number outside the parentheses, the 5, is written only once on the left and is written two times on the right.

$$5 \times (4+3) = 5x4 + 5x3$$

The next example shows the distributive law as it would apply to a sum of three numbers.

$$6 \times (7+4+3) = 6x7 + 6x4 + 6x3$$

This shows that when we have a sum in parentheses and are multiplying by a number outside the parentheses we can simply multiply the outside number times each number of the sum and then add all the products.

The distributive law has a different psychological effect when we switch the left and right sides of the "=" sign. Our original example then looks like:

$$5x4 + 5x3 = 5 \times (4+3)$$

Now the distributive law is telling us how to re-write a sum of products as a single product with a sum. You can do this when the two products have a common number, in this case 5.

Most of beginning algebra is concerned with manipulating symbols. Here we have been manipulating numbers without actually performing any computations. It is important that you be able to read the distributive law both backwards and forwards. We next give two practical computational examples using the distributive law, but keep in mind that the real importance of the distributive law is its ability to manipulate symbols and not its ability to make computations easier.

$$\begin{aligned} 23 \times 103 &= 23 \times (100+3) \\ &= 23x100 + 23x3 \\ &= 2300 + 69 \\ &= 2369 \end{aligned}$$

$$\begin{aligned} 12x17 + 12x13 &= 12 \times (17+13) \\ &= 12 \times 30 \\ &= 3600 \end{aligned}$$

EXERCISES: Use the distributive law to perform the following computations.

Sample 1:
$$\begin{aligned} 6 \times (8+9) &= ? \\ 6 \times (8+9) &= 6x8 + 6x9 \\ &= 48 + 54 \\ &= 102 \end{aligned}$$

Sample 2:
$$\begin{aligned} 13x27 + 13x23 &= ? \\ 13x27 + 13x23 &= 13 \times (27+23) \\ &= 13 \times 50 \\ &= 650 \end{aligned}$$

1. 7 x (4+9)
2. 8 x (6+3)
3. 13 x (4+7)
4. 8 x (5+6)
5. 14 x (5+3)
6. 13x9 + 13x3
7. 7x11 + 7x16
8. 6x9 + 6x14
9. 15x3 + 15
10. 9x5 + 9x15
11. 11 x (4-2)
12. 8x13 - 8x11
13. 15x7 + 15x3
14. 19x3 + 19x17
15. 37x5 + 37x15
16. 4 x (5+8+11)
17. 3 x (2+7+6)
18. 13x3 + 13x5 + 13x7
19. 47x2 + 47x8 + 47x10
20. 51x6 + 51x3 + 51x4

The answers to the previous exercises may be found on page 134.

SECTION I REVIEW PROBLEMS

Perform the indicated operations in problems 1-5.

1. $\begin{array}{r} 256 \\ \times 89 \\ \hline \end{array}$

2. $\begin{array}{r} 7054 \\ \times\ 68 \\ \hline \end{array}$

3. $25\overline{)4823}$

4. $89\overline{)4796}$

5. $632\overline{)498517}$

Test the numbers in problems 6-13 for divisibility by 2,3,5, and 10.

6. 8889

7. 6247

8. 8765

9. 2640915

10. 29536

11. 74934

12. 14290

13. 497920

Compute the powers in problems 14-20. Write your answers as single numbers.

14. $2^3 \times 5^2$

15. $3^2 \times 2^5$

16. $4^2 \times 6^2$

17. $3^2 \times 5^2 \times 11^1$

18. $7^2 \times 5^1 \times 11^2$

19. $34^1 \times 13^2 \times 1^{25}$

20. $5^3 \times 7^2 \times 11^1 \times 0^4$

Compute problems 21-30 using the correct order of operations.

21. $418 - 6 \times 17$

22. $(14-3) \times (27-6)$

23. $5 + 3^2 \times 4$

24. $(2+7)^2 + 2^2 + 7^2$

25. $(113-2x4)^2 - (2+4\cdot5)^2$

26. $(17x0+3) \times (15-2) \div (4^2-3)$

27. $5 \times 12^2 \div 16 + 14$

28. $(((3x2-4)x2+5)x2+6)$

29. $4 \times (8+2x2^3x(13+2)^2)$

30. $3 \times (17+2x(4+3x6) \div 4)$

Use prime numbers to find the greatest common divisors and the least common multiples of the pairs of numbers in problems 31-38.

31. 360 and 1200

32. 900 and 240

33. 1323 and 294

34. 1925 and 455

35. 672 and 1155

36. 680 and 952

37. 325 and 585

38. 1764 and 390

Use the distributive law to perform the computations in problems 39-50.

39. 6 x (7+5)

40. 14 x (2+8)

41. 7 x (3+5+7)

42. 7 x 3 + 7 x 17

43. 13 x 2 + 13 x 18

44. 5 x 2 + 5 x 21 + 5 x 7

45. 19 x 14 + 19 x 16

46. 23 x 7 + 23 x 5 + 23 · 18

47. 4 x (3+11+13)

48. 8 x (9+13+8)

49. 31 x 7 + 31 x 13

50. 87 x 4 + 87 x 9 + 87 x 7

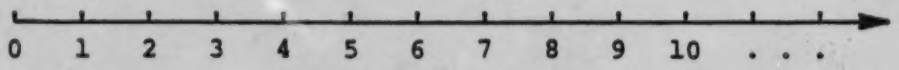
The answers to the Review Problems may be found on page 135.

SECTION II INTEGERS (SIGNED WHOLE NUMBERS)

THE NUMBER LINE

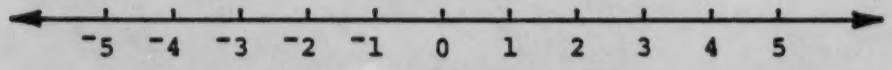
The collection of numbers that we have used so far has consisted of only the whole numbers 0,1,2,3,4,... In this section we will extend our notion of number to include the negative numbers.

There is a nice way to visualize the numbers that we work with which will be familiar to anyone who has used a ruler. The idea is to draw a line and mark points on the line which correspond to the counting numbers. The idea of having numbers correspond to points on a line is exactly the same idea used to make rulers. The picture below is called a number line. Note that the smallest number on this line is zero.



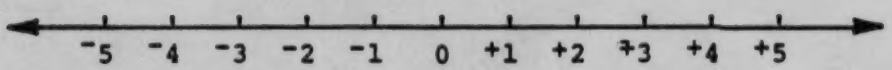
The numbers get larger as we move from left to right on the number line. The arrow pointing to the right indicates that the line continues forever in that direction.

We have written all the counting numbers to the right of zero. Now suppose we wish to move in the opposite direction. What numbers should we use to start counting, going to the left? We simply invent new numbers and we call the new numbers negative numbers. The picture below shows a complete number line with arrows pointing both left and right.



The numbers to the left of zero are composed of a negative sign and a number. Thus, "-1" is read as "negative one" and "-15 is read as "negative fifteen". You don't physically see -15 on the above line, but if our paper were wide enough to show -15, it would be 10 marks to the left of -5.

The old counting numbers together with the new negative numbers are called signed whole numbers, or integers. To contrast the numbers to the right of zero with those to the left we will also call the numbers on the right the positive integers. We use the symbol "+" to denote positive numbers. Thus, "+3" is read as "positive three". Conceptually there is no difference between the numbers that we call the whole numbers and the positive integers. If we place "+" signs on the numbers to the right of zero our number line looks like the following:



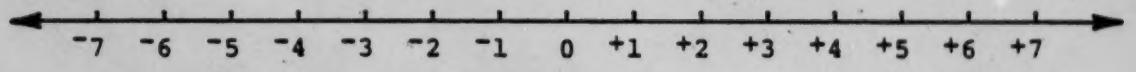
Notice that the number 0 does not have a negative or a positive sign. 0 is a neutral number and is neither positive nor negative. But all other numbers are either positive or negative.

Although the positive sign "+" and the negative sign "-" look like the symbols we use for addition and subtraction, they're not used in the same way. The positive and negative signs are written above the line where the plus and minus signs would normally be written. Thus, $^{-}5$ denotes negative five. We write the negative sign "-" in front of and at the top of 5. When we write positive three the positive sign "+" should be written in front of and at the top of 3; $^{+}3$.

When we write addition or subtraction problems we write the numbers we add or subtract with immediately in front of and immediately behind the addition or subtraction symbols "+" (plus) and "-" (minus). So there should not be any confusion between positive and plus and negative and minus. $^{-}7$ denotes negative 7 but 13-7 denctes 13 minus 7.

You may have already had some experience with negative numbers in learning about temperature scales. If you have ever written a check that bounced you know why negative numbers are important. Businesses which report profits and losses also use negative numbers. In algebra negative numbers are used to solve equations.

OPPOSITES



If we take another look at the number line we can see how the positive and negative numbers are evenly divided by the number 0 which is neither positive nor negative. $^{+}2$ and $^{-}2$ are on opposite sides of 0. For this reason we say the opposite of $^{+}2$ is $^{-}2$ and we say the opposite of $^{-}2$ is $^{+}2$.

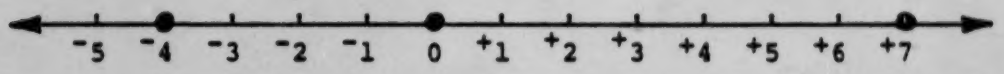
In algebra, if x represents a number then the notation -x denotes the opposite of x. "-x" is read "the opposite of x". Thus, "$-(^{-}2)$" is read as "the opposite of negative two" and $-(^{-}2) = +2$. "$-(^{+}2)$" is read as "the opposite of positive two" and $-(^{+}2) = -2$. The sign for the opposite of "-" looks like the subtraction symbol minus. However, just as with negatives, we can tell the difference by checking whether there is a number immediately in front of "-". If so, then "-" means subtract. But if there is no number immediately in front of "-", then this symbol means "the opposite of".

Some books don't distinguish between negatives and opposites. In algebra it's important to realize that -x does not mean "negative x". -x may denote a positive number. What you see is not what you get. In fact, depending on the value of x, -x may be positive, or zero, or negative. For instance, if $x = (^{-}2)$ then $-x = -(^{-}2) = ^{+}2$.

ABSOLUTE VALUE

Once we have a geometric picture for our number system we can ask the question, "How do you find the distance between any two points on the number line?" The concept of absolute value will help us answer this question. Remember that distances are never negative.

The absolute value of a number is the distance between the points on the number line that correspond to the given number and the number 0. The absolute value of a positive number is just the number. The absolute value of a negative number is the positive number that results from dropping the negative sign.



Now look at the above number line. The distance between 0 and $^{+}7$ is 7 units. The distance between $^{-}4$ and 0 is 4. It is customary to write positive numbers without the "+" sign, but if we wish to emphasize that a number is positive then we can do so by writing "+" in front of the number. Negative numbers must always be written with the "-" sign immediately in front of the number.

In algebra we use letters of the alphabet to represent numbers. In the future, "x" will not mean multiply.

Since "x" no longer means multiplication (x represts a number) we need another symbol for multiplication. In algebra we can use a dot between two numbers to indicate multiplication. This dot is not a decimal point, it is written above the line. $3 \cdot 5 = 15$. If y represents another number we can indicate the product of x and y by writing, $x \cdot y$. Many times with letters we even leave out the dot and assume that when two letters are written together we should think of multiplication as the operation between the letters. When writing actual numbers we must use the dot. Otherwise we douldn't tell the difference between $3 \cdot 17$ and 317. $3 \cdot 17 = 51$ but 317 is just a single number.

The special notation "$|x|$" is read as "the absolute value of x." When we compute the absolute value of a number we drop the vertical bars and write our answer on the right side of the "=" sign. $|x|$ is the distance between the points on the number line corresponding to x and 0. This distance is never negative.

Examples:

$|+3| = 3$ $\quad$ $|+6| = 6$

$|-2| = 2$ $\quad$ $|-1| = 1$

$|-5| = 5$ $\quad$ $|0| = 0$ $\quad$ why?

If there is a single number inside the absolute value bars, then their effect is to drop the sign of the number. As previously mentioned, we are not required to write the "+" sign on positive numbers but there is no harm in doing so. Thus, we could have written:

$|+3| = +3$ and $|-2| = +2$

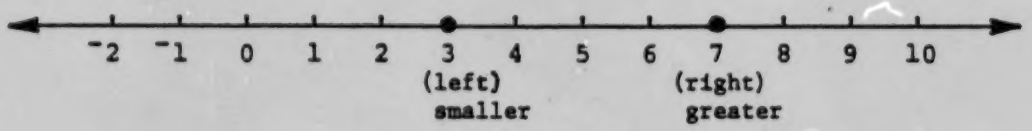
Absolute value makes the sign of nonzero numbers always come out as "+".

Absolute value bars also act as grouping symbols. When more than one number appears between the vertical bars we should perform all operations inside the bars to obtain a single number. Then and only then can we drop the bars and write our answer as a nonnegative number.

ORDER

Numbers are used for comparing quantities. In mathematics the symbols ">" and "<" stand for "greater than" and "less than". These notations are used as shortcuts. Instead of writing 8 is greater than 5, we use the symbol ">" and write 8 > 5. Since 5 is less than 8, we could also write 5 < 8. We can always tell which number is larger and which is smaller because both of the symbols > and < point in the direction of the smaller number.

"3 < 7" is read "three is less than seven" and "7 > 3" is read "seven is greater than three." Both sentences convey the same message. If we look at these numbers on the number line we see that the larger number lies to the right of the smaller number.



We will take this geometric notion as a definition of greater than or less than for all numbers, including the negative numbers. The number x is defined to be less than the number y, if on the number line, the point corresponding to x lies to the left of the point corresponding to y.

According to this definition, $^{-}1 < {}^{+}4$ since $^{-}1$ lies to the left of $^{+}4$. About the only times the definition becomes confusing is when we write statements like: $^{-}6 < {}^{-}2$ or $^{-}6 < {}^{+}1$. Of course with positive numbers we know $^{+}6 > {}^{+}2$ and $^{+}6 > {}^{+}1$. But when negative numbers are involved, we must think of a picture of the number line. The rule to be followed is that smaller numbers are on the left, or another way of saying the same thing, larger numbers are on the right.

Example: Insert only one of the symbols > or < between the following pairs of numbers to make true statements.

5 ? 9	5 > 9	9 lies to the right of 5.
$^{-}4$? 3	$^{-}4 < 3$	$^{-}4$ lies to the left of $^{+}3$.
6 ? $^{-}10$	6 > $^{-}10$	6 lies to the right of $^{-}10$.

Whenever in doubt as to how the comparison should go, think of a picture of the number line and make your symbol point towards the number that is on the left on the number line.

EXERCISES: Compute the absolute values in problems 1-15. Insert one of the symbols > or < between the pairs of numbers in problems 16-25 to make true statements.

Samples:		
	$\lvert^{-}3\rvert$ = ?	$\lvert^{-}3\rvert = {}^{+}3$
	$\lvert^{+}12\rvert$ = ?	$\lvert^{+}12\rvert = {}^{+}12$
	$\lvert 18-2\cdot 6\rvert$ = ?	$\lvert 18-2\cdot 6\rvert = \lvert 18-12\rvert = \lvert 6\rvert = {}^{+}6$
	5 ? $^{-}1$	5 > $^{-}1$ $^{-}1$ is on the left.

$^-10$? $^-8$ $^-10 < ^-8$ $^-10$ is on the left.

1. $|^-4| =$ 2. $|^+4| =$ 3. $|4| =$ 4. $|^-17| =$
5. $|^+13| =$ 6. $|^-6| =$ 7. $|-9| =$ 8. $|^-12| =$
9. $|^-8| =$ 10. $|11| =$ 11. $|8+3| =$ 12. $|8-3| =$
13. $|7+3\cdot5| =$ 14. $|3\cdot(18-7)| =$ 15. $|13-3\cdot2+4^2| =$

16. 3 ? 5 17. 3 ? $^-5$ 18. $^-12$? $^-6$
19. $^-1$? 10 20. 10 ? $^-1$ 21. $^-20$? $^-25$
22. 33 ? $^-8$ 23. $^-21$? $^-22$ 24. 5 ? $^-6$
25. $^-5$? $^-7$

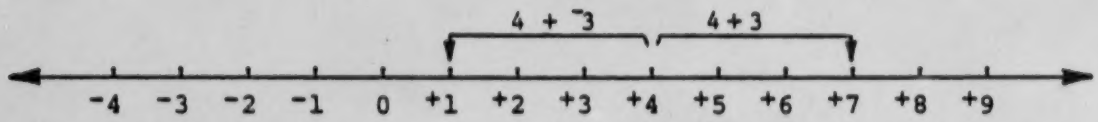
The answers to the previous exercises may be found on page 136.

ADDITION

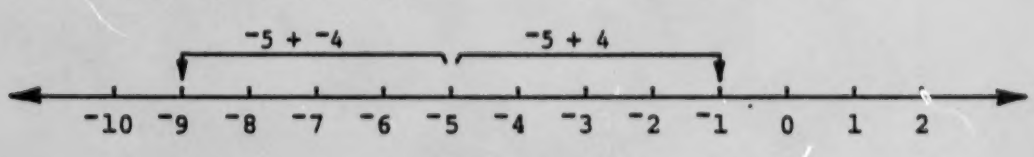
In this section we will learn how to add signed numbers. In the following two sections we will work on subtraction, multiplication, and division.

Let's look at our number line as a way of visualizing what happens when we perform addition of numbers. $4 + 3 = 7$. If we start at 4 on the line and then add 3 the effect is to move 3 units to the right. We move to the right because $3 = {}^{+}3$. Now what do you think we should do if we start at 4 and add $^{-}3$?

When we add a positive number we move to the right, so when we add a negative number we should move to the left. Thus, $4 + {}^{-}3$ means start at positive 4 on the number line and move 3 units to the left.
$4 + {}^{-}3 = {}^{+}1$



Now we'll look at two more examples of addition on the number line. $^{-}5 + 4 = {}^{-}1$ and $^{-}5 + {}^{-}4 = {}^{-}9$.



In each example we are adding a number to $^{-}5$. Each problem begins with $^{-}5$ and so we should begin at $^{-}5$ on the number line. The first problem, $^{-}5 + 4$, adds positive 4 to negative 5, so we should move from $^{-}5$, 4 units to the right. We end up at $^{-}1$ which is the correct answer. The second problem, $^{-}5 + {}^{-}4$, adds negative 4 to negative 5. We again start on the number line at $^{-}5$, but this time we move left 4 units. Now we end up at $^{-}9$ which is the correct answer.

When we add numbers on the number line, the point we start at should be the first number. We then move a distance from the first number equal to the second number. The sign of the second number tells us whether we move left or right. If the second number is positive we move right. If the second number is negative we move left.

These rules for adding numbers on a number line will always give the correct answer. But they have one drawback. In order to add $273 + {}^{-}584$ we would have to draw a very long number line which would require a rather big piece of paper. A number line can help us visualize a problem but there are other ways of working problems which avoid drawing pictures.

To add two signed whole numbers:

1) If both numbers are positive the answer will be positive and the numerical value is the ordinary sum of the numbers.

2) If both numbers are negative the answer will be negative and

the numerical value is the ordinary sum of the numbers (without signs).

3) If one number is positive and the other is negative then the sign of the answer will be the sign of the "larger" number. The numerical value is the ordinary difference of the numbers (without signs) obtained by subtracting the smaller from the larger.

Examples: $27 + 53$ This is ordinary addition. $27 + 53 = {}^{+}80$

$^{-}13 + {}^{-}19$ Since both numbers are negative the answer is negative. The numerical value is $13 + 19 = 32$. $^{-}13 + {}^{-}19 = {}^{-}32$

$^{-}36 + 14$ The first number is negative, the second is positive. 36 is larger than 14 so the sign of the answer is the sign of $^{-}36$. The numerical value is $36 - 14 = 22$, $^{-}36 + 14 = {}^{-}22$.

$63 + {}^{-}59$ The first number is positive; the second number is negative. 63 is larger than 59, so the sign of the answer is the sign of 63. The numerical value is $63 - 59 = 4$, $63 + {}^{-}59 = {}^{+}4$.

$72 + {}^{-}103$ The first number is positive, the second number is negative. 103 is larger than 72 so the sign of the answer is the sign of $^{-}103$. The numerical value is $103 - 72 = 31$, $72 + {}^{-}103 = {}^{-}31$.

EXERCISES: Add the following signed numbers.

1. $56 + {}^{-}39$	2. $^{-}16 + {}^{-}73$	3. $^{-}58 + 75$
4. $^{-}66 + {}^{-}107$	5. $93 + {}^{-}44$	6. $^{-}94 + 35$
7. $^{-}83 + {}^{-}145$	8. $^{-}97 + 196$	9. $291 + {}^{-}187$
10. $129 + {}^{-}480$	11. $^{-}422 + 247$	12. $^{-}544 + 437$
13. $^{-}792 + {}^{-}731$	14. $^{-}117 + 476$	15. $703 + {}^{-}911$
16. $504 + {}^{-}244$	17. $647 + {}^{-}936$	18. $127 + {}^{-}128$
19. $^{-}487 + 487$	20. $^{-}353 + 143$	21. $208 + {}^{-}593$

22. $103 + {}^{-}693 + 83$

23. $^{-}97 + (861 + {}^{-}952) + 757$

24. $|{}^{-}23+46+{}^{-}80| + |{}^{-}411+257|$

25. $|{}^{-}245+302+{}^{-}964| + |198+{}^{-}697+493|$

The answers to the previous exercises may be found on page 136.

SUBTRACTION

If you have mastered addition of signed numbers then there is only one additional rule to learn to do subtraction. Every subtraction problem can be turned into an addition problem with the following rule.

To subtract one signed number from another:

Change the sign of the second number which is being subtracted and then add the two numbers using the addition rules.

Examples:

$^{-}36 - 48$ — The second number which is being subtracted is positive 48. We change the sign to negative and add. $^{-}36 - 48 = {}^{-}36 + {}^{-}48 = {}^{-}84$

$72 - {}^{-}15$ — The second number which is being subtracted is negative 15. We change the sign to positive and add. $72 - {}^{-}15 = 72 + {}^{+}15 = {}^{+}87$

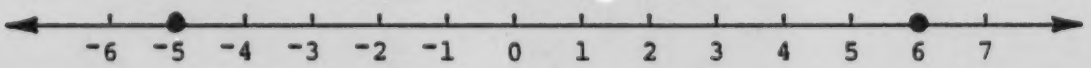
$^{-}528 - {}^{-}324$ — The second number which is being subtracted is negative 324. We change the sign to positive and change the subtraction to addition. $^{-}528 - {}^{-}324 = {}^{-}528 + {}^{+}324 = {}^{-}204$

$17 - 12$ — The number being subtracted is positive 12. We change the sign to negative and add. $17 - 12 = 17 + {}^{-}12 = {}^{+}5$
Note that the problem $17 - 12$ is just ordinary subtraction and we get the same answer when thinking of this as a problem with positive and negative numbers.

Now that we have learned how to subtract positive and negative numbers we can take another look at absolute value. When we first introduced this concept we asked the question "How do you find the distance between any two points on the number line?" The answer is simple. If the points correspond to the numbers x and y then the distance between the two points is $|x-y|$ or $|y-x|$. Both absolute values are equal to each other and we may subtract in either order as long as we finish by taking the absolute value.

The next two examples illustrate how this works.

Examples: What is the distance between the points corresponding to $^{-}5$ and $^{+}6$?



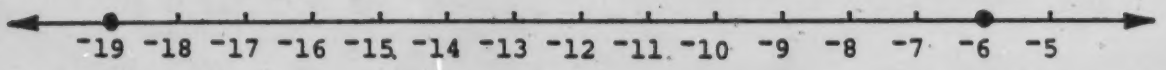
$|{}^{-}5 - 6| = |{}^{-}5 + {}^{-}6| = |{}^{-}11| = {}^{+}11$

or

$|6 - {}^{-}5| = |6 + 5| = |{}^{+}11| = {}^{+}11$

The distance is $^{+}11$.

What is the distance between the points corresponding to $^{-}19$ and $^{-}6$?



$|^{-}19 - {}^{-}6| = |^{-}19 + {}^{+}6| = |^{-}13| = {}^{+}13$

$|^{-}6 - {}^{-}19| = |^{-}6 + {}^{+}19| = |{}^{+}13| = {}^{+}13$

The distance is $^{+}13$.

EXERCISES: Subtract the following signed numbers.

1. $^{-}13 - 9$
2. $27 - 38$
3. $^{-}31 - 67$
4. $12 - {}^{-}4$
5. $^{-}11 - {}^{-}13$
6. $64 - 84$
7. $^{-}56 - 14$
8. $^{-}127 - {}^{-}5$
9. $^{-}42 - 27$
10. $118 - 232$
11. $322 - {}^{-}638$
12. $^{-}988 - {}^{-}366$
13. $563 - 582$
14. $894 - {}^{-}700$
15. $^{-}330 - 874$
16. $400 - {}^{-}863$
17. $193 - 227$
18. $^{-}118 - 2395$
19. $27 - (13-19)$
20. $284 - (806-689)$
21. $546 - (938-(589-597))$
22. $266 - ({}^{-}741 - ({}^{-}235-{}^{-}458))$
23. $786 - ({}^{-}457-(808-{}^{-}715)-{}^{-}389)$
24. $|236-512|- |357-411|$
25. $|17+{}^{-}31| -|{}^{-}86+13| - |{}^{-}10-14|$
26. A tall building has 10 floors above ground and 3 floors below ground. If you start on the 5th floor above ground and go down 7 floors and then up 1 floor, where do you get off at?
27. An airplane is flying at 25,000 feet and then descends 7000 feet and later climbs 2000 feet. What is the present altitude?
28. A submarine dives 530 feet below sea level and then climbs 115 feet and then dives 75 feet. What is the present position of the submarine?
29. The temperature measured in degrees farenheit shows 48^{0} before a cold snap hits which lowers the temperature 36^{0}. A second cold snap hits and again changes the temperature by 17^{0}. What is the final temperature?
30. Your checking account has a balance of $109 before you write checks for $19, $32, $45 and $37. What is your final new balance? Should you get a checking account with overdraft protection?

31. What is the distance between the points on the number line that correspond to -21 and 53?

32. What is the distance between the points on the number line that correspond to -317 and 512?

The answers to the previous exercises may be found on page 136.

MULTIPLICATION AND DIVISION

The rules for multiplication and division of signed numbers are simpler than the rules for addition and subtraction. It is important not to confuse the rules for these four basic operations. None of the rules by themselves are difficult. Each time you do an operation you need to figure two quantities. One quantity is the sign of the answer and the other is the numerical value. The rules for multiplication and division are:

1) The numerical value of the answer is the number obtained by ordinary multiplication or division.
2) If the two numbers have the same sign (both are positive or both are negative) then the sign of the answer is positive.
3) If the two numbers have opposite signs (one is positive and one is negative) then the sign of the answer is negative.

Many times we enclose negative numbers within parentheses to make problems more readable. Thus, $(-3) \cdot (-6)$ means multiply negative 3 times negative 6. The answer to this problem according to the above rules is $^{+}18$. As you simplify longer problems you may also wish to enclose negative numbers within parentheses. Doing so helps avoid any confusion between the negative sign and subtraction.

Examples:

$(-16) \div (8) = {}^{-}2$.	Ordinary division (without regard to signs) gives the numerical answer 2. Since these two numbers have opposite signs the sign of the answer is negative.
$(16) \cdot ({}^{-}8) = {}^{-}128$	Same reasoning as above except we multiply instead of divide.
$({}^{-}5) \cdot ({}^{-}4) = {}^{+}20$	Both numbers have the same sign so the answer will be positive.
$({}^{-}42) \div ({}^{-}6) = {}^{+}7$	Both numbers have the same sign so the answer is positive.

Before working the problems below you may wish to go back and review the rules for addition and subtraction. When multiplying or dividing signed numbers the operation involved is always ordinary multiplication or ordinary division to obtain the numerical part of the answer. But when adding signed numbers, sometimes we subtract to obtain the numerical part of the answer. When subtracting signed numbers sometimes we end up performing ordinary addition. The rule of signs for multiplication and division is different from the rule of signs for addition in two respects. Adding two negatives always gives a negative answer. When adding two numbers with different signs (one positive and one negative) you can't tell what the sign of the answer should be based on the signs of the two numbers. You must take into account the relative sizes of the numbers. Practice is probably the best way to learn all of these rules and to not confuse the rules for multiplication and addition. You may also wish to review the order of operations.

EXERCISES: Solve the following problems which involve multiplication and division as well as addition and subtraction of signed numbers.

Sample: $(4-18)\cdot(^{-}3)^2$

$= (4+^{-}18)\cdot(9)$

$= (^{-}14)\cdot(9)$

$= ^{-}126$

1. $(^{-}3)\cdot(^{-}4)$
2. $15\div(^{-}3)$
3. $(^{-}76)\div(^{-}38)$
4. $(^{-}15)\cdot(3)$
5. $(^{-}96)\div 6$
6. $(^{-}12)\cdot(^{-}3)$
7. $(^{-}51)\div 17$
8. $13\cdot(^{-}4)$
9. $(^{-}16)\cdot(^{-}4)$
10. $(^{-}2)\cdot(^{-}1+^{-}8)$
11. $(^{-}13)\cdot 5 + (^{-}13)\cdot(^{-}5)$
12. $^{-}4 + (^{-}4)\cdot 15$
13. $5\cdot 3 + 5\cdot(^{-}6)$
14. $(^{-}7)\cdot(^{-}2)\cdot(^{-}5)$
15. $(^{-}14)\cdot(13+^{-}8)$
16. $(^{-}6)\cdot|^{-}4|$
17. $(|^{-}8|+ 3)\cdot(^{-}5)$
18. $^{-}8 + (^{-}24)\div 6$
19. $5\cdot(^{-}7) + 27 \div (^{-}9)$
20. $(24-25)\cdot(16-23)$
21. $(7-17\div 6)\cdot(26\div(^{-}13)+(^{-}4))$
22. $120 - (13-16)\cdot(^{-}6)^2$
23. $(^{-}4)^2+ (^{-}3)^2$
24. $(^{-}4+^{-}3)^2$
25. $(21-9)\cdot(11-13)\cdot(5)\div(^{-}60)$
26. $((7\cdot(^{-}2)+3)\cdot(^{-}2)-15)\cdot(^{-}2)$
27. $((^{-}3+5)\cdot(4)-12)\cdot(4) + 5$
28. $|^{-}8|\cdot(^{-}4) + |^{-}10|\cdot(6) + (^{-}4)^2$
29. $((^{-}5)^2-(^{-}7)^2)^2$
30. $((^{-}5-(^{-}7))^2)^2$

The answers to the previous exercises may be found on page 137.

SECTION II REVIEW PROBLEMS

Insert one of the symbols > or < between the pairs of numbers in problems 1-10 to make true statements.

1. $-4 \;?\; 3$
2. $5 \;?\; {}^{-}6$
3. ${}^{-}8 \;?\; {}^{-}7$
4. ${}^{-}8 \;?\; {}^{-}9$
5. ${}^{-}25 \;?\; 24$
6. $23 \;?\; {}^{-}24$
7. ${}^{-}12 \;?\; 2$
8. $258 \;?\; {}^{-}459$
9. ${}^{-}37 \;?\; 0$
10. $26 \;?\; {}^{-}43$

Perform the indicated operations on the signed numbers in problems 11-50. Use the correct order of operations.

11. $36 + {}^{-}39$
12. $24 - 30$
13. ${}^{-}17 + 27$
14. ${}^{-}12 + {}^{-}6$
15. $15 - {}^{-}7$
16. ${}^{-}12 - 20$
17. $618 + {}^{-}324$
18. ${}^{-}549 + 252$
19. $42 - {}^{-}24$
20. ${}^{-}58 - {}^{-}29$
21. $|{}^{-}36 + 6|$
22. $|28 - 39|$
23. $|{}^{-}4 + {}^{-}5|$
24. $|13 + {}^{-}8|$
25. $|{}^{-}18 + 10|$
26. $({}^{-}5) \cdot ({}^{-}3)$
27. $24 \div ({}^{-}6)$
28. $({}^{-}39) \div ({}^{-}13)$
29. $({}^{-}17) \cdot 3$
30. ${}^{-}12 + 3 - 8$
31. $(-14) \cdot ({}^{-}3) + ({}^{-}9)$
32. $({}^{-}5) + ({}^{-}60) \div ({}^{-}4)$
33. $32 - (6) \cdot (-8)$
34. $6 - (8-(10-12))$
35. ${}^{-}3 + ({}^{-}5-({}^{-}7+{}^{-}9))$
36. $(-4) \cdot (3+({}^{-}17-5))$
37. $16 - (25-(36-49))$
38. $({}^{-}5+2)^2 - (6-10)^2$
39. $({}^{-}5) \cdot (18+(-3) \cdot (8+({}^{-}2) \cdot (7)))$
40. $6 \cdot (-8)^2 \div (-17+5) - (3 \cdot 0+{}^{-}8)$

41. $((^{-}16) + (^{-}4)\cdot(^{-}3)^2)^2 - ((^{-}9)\cdot(5) - ^{-}32)^2$

42. $(((6\cdot(^{-}3)-5)\cdot(^{-}2)+7)\cdot(-1))\cdot(-2)-12$

43. $(^{-}80 \div ((^{-}50) \div (30\div((^{-}12)\div(2))))$

44. $(^{-}3) \cdot (13-(^{-}2)\cdot(3-2\cdot(8-15)))$

45. $((-4)\cdot((-5)\cdot 2+6) + ^{-}13) \cdot (-14) + (-15)$

46. $4\cdot(^{-}2)^3 + (4\cdot(^{-}2))^3$

47. $(^{-}2)\cdot(8+(^{-}2)\cdot(9+(^{-}2)\cdot(17-(^{-}2)\cdot((^{-}2)\cdot 5-11))))$

48. $(-2)^2\cdot(10+(-2)^3\cdot(15+(-2)^3\cdot(13+(-2)^3)))$

49. $((((^{-}3)\cdot 7-5)\cdot(^{-}3)+4)\cdot(-3)+8)\cdot(^{-}3)+13$

50. $-5 \cdot 10^4 + ^{-}3\cdot 10^3 + -7\cdot 10^2 + -4\cdot 10 + -8$

The answers to the Review Problems may be found on page 138.

SECTION III FRACTIONS

FUNDAMENTAL PROPERTY

A fraction is the quotient of two integers. If m and n denote the two integers then the symbol for the fraction using m and n is $\frac{m}{n}$, read as "m over n". The top number in a fraction is called the numerator, the bottom number is called the denominator. The denominator of a fraction can never be zero.

Fractions are needed to subdivide the standard units. Any whole quantity can be broken down into smaller units and fractions provide one way of describing parts of a whole. We previously marked points on the number line for the integers, and using fractions will help us fill in some of the gaps between the whole numbers. In a later section we will study decimals which also fill in some of the points that are missing from the number line. We assume this is not your first introduction to fractions. But if you haven't worked with fractions in a while then you will want to review all of the rules that we present here.

Some of the skills to be developed in working with fractions include the basic four operations of addition, subtraction, multiplication, and division, as well as being able to compare fractions. Your ability to use these basic skills depends on what we will call the Fundamental Property of Fractions. This property tells how to expand or reduce a fraction.

Fundamental Property of Fractions: Both the numerator and denominator of a fraction may be multiplied or divided by the same nonzero number without changing the value of the fraction.

Multiplication would normally be used to expand a fraction and division can be used to reduce a fraction. For example, to reduce the fraction $\frac{9}{21}$ divide both numerator and denominator by 3. The Fundamental Property guarantees that we won't change the value of the fraction, although we certainly will be changing its form.

$$\frac{9}{21} = \frac{9\div3}{21\div3} = \frac{3}{7}$$

The reason we chose to divide by 3 was because 3 divides evenly into both 9 and 21. When reducing fractions we may find it necessary to apply the Fundamental Property several times. For example, to reduce $\frac{98}{154}$ we may immediately recognize that both numbers are even and therefore divisible by 2. So, $\frac{98}{154} = \frac{98\div2}{154\div2} = \frac{49}{77}$. Next we may recognize that both numbers in this last fraction are evenly divisible by 7. $\frac{49}{77} = \frac{49\div7}{77\div7} = \frac{7}{11}$. The final result is that $\frac{98}{154} = \frac{7}{11}$. We could have obtained this answer in one step had we known both numbers could be divided by 14.

$$\frac{98}{154} = \frac{98\div14}{154\div14} = \frac{7}{11} .$$

All of the mystery of how to reduce fractions can be eliminated by using the prime numbers. The primes tell you what numbers to divide with and they also tell you when you are finished. One way to reduce a fraction $\frac{m}{n}$, is to first compute the greatest common divisor (GCD) of m and n. Then divide both m and n by their GCD. The resulting fraction will be completely reduced.

Of course when we compute the GCD we have to find the prime divisors of both numbers. A shorter procedure than the one above is to go down the list of prime numbers until you find a prime that divides evenly into both the numerator and denominator. Then apply the Fundamental Property. The fraction will be completely reduced after you have divided by all the primes common to both numerator and denominator. You may wish to go back and review the section on prime numbers.

Example: Reduce $\frac{4290}{39270}$

$$\frac{4290}{39270} = \frac{4290\div2}{39270\div2} = \frac{2145}{19635} = \frac{2145\div3}{19635\div3} = \frac{715}{6545} =$$

$$= \frac{715\div5}{6545\div5} = \frac{143}{1309} = \frac{143\div11}{1309\div11} = \frac{13}{119}$$

The process of expanding a fraction is just the opposite of reducing a fraction. Most practical problems involve multiplying by a number to obtain a least common denominator.

Example: Expand the fraction $\frac{2}{3}$ to obtain a fraction whose denominator is 24.

To expand the given denominator 3 to make it equal to 24 we would multiply by 8. But the Fundamental Principle tells us we must also multiply the numerator by 8.

$$\frac{2}{3} = \frac{2\cdot8}{3\cdot8} = \frac{16}{24}$$

EXERCISES: Apply the Fundamental Principle of Fractions to solve the following problems.

Sample 1: Reduce $\frac{510}{570}$

Use the list of prime numbers to find the primes which divide evenly into both 510 and 570.

$$\frac{510}{570} = \frac{510\div2}{570\div2} = \frac{255}{285} = \frac{255\div3}{285\div3} = \frac{85}{95} = \frac{85\div5}{95\div5} = \frac{17}{19}$$

Sample 2: Write $\frac{2}{5}$ as a fraction with a denominator of 35.

Compare the given denominator 5 with 35. Multiplying 5 by 7 will make 35. So we should multiply both 2 and

5 by 7.

$\frac{2}{5} = \frac{2 \cdot 7}{5 \cdot 7} = \frac{14}{35}$ $\frac{14}{35}$ = is another name for $\frac{2}{5}$.

Reduce:

1. $\frac{24}{27}$
2. $\frac{75}{105}$
3. $\frac{42}{105}$
4. $\frac{42}{78}$
5. $\frac{34}{102}$
6. $\frac{30}{129}$
7. $\frac{36}{198}$
8. $\frac{77}{245}$
9. $\frac{160}{450}$
10. $\frac{72}{174}$
11. $\frac{42}{518}$
12. $\frac{35}{147}$
13. $\frac{210}{270}$
14. $\frac{280}{175}$
15. $\frac{39}{33}$
16. $\frac{78}{66}$
17. $\frac{143}{234}$
18. $\frac{308}{1155}$
19. $\frac{1870}{4180}$
20. $\frac{273}{924}$

Expand each fraction in problems 21-30 to an equivalent fraction with the indicated denominator.

21. $\frac{18}{23}$ denominator = 69
22. $\frac{5}{13}$ denominator = 78
23. $\frac{7}{12}$ denominator = 96
24. $\frac{7}{4}$ denominator = 32
25. $\frac{11}{18}$ denominator = 126
26. $\frac{5}{41}$ denominator = 205
27. $\frac{11}{8}$ denominator = 168
28. $\frac{9}{16}$ denominator = 96
29. $\frac{17}{48}$ denominator = 384
30. $\frac{25}{28}$ denominator = 84

The answers to the previous exercises may be found on page 139.

FRACTIONS AND MIXED NUMBERS

When we expand or reduce fractions we obtain answers that are equal (but in a different form) to the fractions we started with. There is a simple trick which can be used to tell when two fractions are equal. The trick is called cross multiplying and works as follows. Draw an X over the equals symbol.

$$\frac{2}{3} \times \frac{8}{12}$$

Then multiply the pairs of numbers connected by the lines in the X. This amounts to multiplying the numbers in the upper left and lower right positions, and multiplying the numbers in the lower left and upper right positions. Then test if these two products are equal.

$$2 \cdot 12 \overset{?}{=} 3 \cdot 8$$

$$24 = 24$$

For this example the cross products are equal and therefore the original fractions were also equal.

Example: Is $\frac{3}{13} = \frac{4}{17}$?

Cross multiply to check

$3 \cdot 17 = 51$ and $13 \cdot 4 = 52$

Since $51 \neq 52$, $\frac{3}{13} \neq \frac{4}{17}$

When we use negative numbers in fractions we can simplify our answers by keeping the number of negative signs to a minimum. Since a negative number divided by a negative number gives a positive answer, we may write $\frac{-2}{-3} = \frac{2}{3}$. We never need to write more than one negative sign in our final answer, and of course if our final answer is a positive number then we don't need any negative signs at all.

It is important to realize that an answer which requires a negative sign may be written in one of three forms. For example:

$$\frac{-3}{5} = \frac{3}{-5} = -\frac{3}{5}$$

The negative sign may be written in the numerator, or in the denominator, or in front of the entire fraction.

Example: Write $-\left(\frac{-3}{-4}\right)$ using a minimum number of negative signs.

$$-\left(\frac{-3}{-4}\right) = -\frac{3}{4}$$

It is customary to write the negative sign in front of the fraction or in the numerator, rather than in the denominator.

We previously defined a fraction as a quotient of two integers.

In fact, a quotient of any two integers is always a fraction with one important exception. The denominator of a fraction cannot be the number zero.

You can't divide by 0

The reason the above statement holds is that when we divide, the remainder should be smaller than the divisor. Try dividing 0 into 5 to see why.

$$0\overline{)5}\quad \begin{matrix} ? \\ 0 \\ \hline 5 \end{matrix}$$

No matter what digit you put in the quotient, when you multiply by the divisor 0 you get 0 and 0 subtracted from 5 is 5. The remainder 5 is larger than the divisor which is 0. Usually when the remainder is larger than the divisor it means you didn't choose a large enought digit in the quotient. In this case though it doesn't matter what digit you put in the quotient since zero times any number is always zero. Division by zero just doesn't work!

So we can never have 0 for a denominator of a fraction. But we can have 0 in the numerator of a fraction.

$\frac{5}{0}$ is undefined. $\frac{0}{5}$ is a valid fraction.

You can't divide by 0 but you can divide into 0. For example:

$$5\overline{)0}\quad \begin{matrix} 0 \\ \hline 0 \end{matrix}$$

This shows that $\frac{0}{5} = 0$.

A positive fraction in which the numerator is larger than the denominator is called an improper fraction. Examples of improper fractions are:

$$\frac{7}{4} \qquad \frac{10}{3} \qquad \frac{37}{11}$$

Improper fractions may be changed to mixed numbers which involve a whole number together with a fraction.

$$\frac{7}{4} = 1\frac{3}{4} \qquad \frac{10}{3} = 3\frac{1}{3}$$

The whole number part is the quotient when the denominator is divided into the numerator. The remainder then forms the new numerator of the fractional part. For example, dividing 11 into 37 yields:

$$11\overline{)37}\quad \begin{matrix}3\\ \frac{33}{4}\end{matrix} \qquad \text{so,} \quad \frac{37}{11} = 3\frac{4}{11}$$

The real meaning of $3\frac{4}{11}$ is $3 + \frac{4}{11}$.

When we convert a mixed number back into an improper fraction we multiply and add just as we do in performing the "division check."

$$4\frac{3}{5} = \frac{23}{5} \qquad \text{since,} \quad 4 \cdot 5 + 3 = 23$$

A negative sign written in front of a mixed number applies to both the whole number and the fractional parts. Thus, $-\frac{23}{6} = -3\frac{5}{6}$ which means $-3 + -\frac{5}{6}$.

EXERCISES:

In problems 1-10 tell whether or not the two fractions are equal by cross multiplying.

1. $\frac{3}{4} \stackrel{?}{=} \frac{7}{12}$
2. $\frac{3}{4} \stackrel{?}{=} \frac{6}{8}$
3. $\frac{3}{7} \stackrel{?}{=} \frac{4}{9}$
4. $\frac{9}{5} \stackrel{?}{=} \frac{7}{4}$
5. $\frac{7}{8} \stackrel{?}{=} \frac{6}{7}$
6. $\frac{2}{34} \stackrel{?}{=} \frac{-3}{51}$
7. $\frac{15}{35} \stackrel{?}{=} \frac{18}{42}$
8. $\frac{-36}{24} \stackrel{?}{=} \frac{6}{-4}$
9. $\frac{7}{13} \stackrel{?}{=} \frac{6}{11}$
10. $\frac{3}{37} \stackrel{?}{=} \frac{4}{49}$

Change the improper fractions in problems 11-20 to mixed numbers.

11. $\frac{13}{6}$
12. $\frac{29}{8}$
13. $\frac{-35}{13}$
14. $\frac{-49}{6}$
15. $\frac{126}{49}$
16. $\frac{-232}{27}$
17. $\frac{-258}{39}$
18. $\frac{117}{19}$
19. $\frac{143}{13}$
20. $\frac{-153}{17}$

In problems 21-30 convert the mixed numbers into improper fractions.

21. $2\frac{3}{4}$
22. $5\frac{5}{7}$
23. $6\frac{7}{8}$
24. $-5\frac{1}{6}$
25. $-8\frac{2}{3}$
26. $11\frac{12}{13}$
27. $-4\frac{5}{9}$
28. $-13\frac{2}{5}$

29. $-27\frac{1}{2}$ 30. $32\frac{1}{5}$

The answers to the previous exercises may be found on page 139.

PROPORTIONS

The quotient of two numbers is called a ratio and the fact that two ratios are equal is called a proportion. The statement $\frac{2}{3} = \frac{8}{12}$ is an example of a proportion. In many problems we are missing one of the four numbers in a proportion.

Example: Find the missing number in the proportion.

$$\frac{3}{4} = \frac{?}{24}$$

In this example a numerator is missing but both denominators are present. By comparing the two denominators we can see that the complete fraction $\frac{3}{4}$ should be expanded to make its denominator equal to 24. We can do this by applying the Fundamental Property of fractions and multiplying by 6.

$\frac{3}{4} = \frac{3 \cdot 6}{4 \cdot 6} = \frac{18}{24}$. The missing number is 18.

Example: Find the missing number in the proportion.

$$\frac{-9}{?} = \frac{18}{36}$$

In this example a denominator is missing so we should compare numerators. We always begin with the fraction that is complete and alter it by applying the Fundamental Property. In this case we begin with 18 and try to convert 18 to -9 by either multiplying or dividing by some number. The same operation with that number applied to 36 will give us the missing denominator. Dividing 18 by -2 yields -9; so, we do the same to 36.

$\frac{18}{36} = \frac{18 \div -2}{36 \div -2} = \frac{-9}{-18}$. The missing number is -18.

EXERCISES: Find the missing numbers in the following proportions.

Sample: $\frac{?}{6} = \frac{35}{42}$

Compare 42 with 6. $42 \div 7 = 6$

$\frac{35}{42} = \frac{35 \div 7}{42 \div 7} = \frac{5}{6}$ Answer = 5.

1. $\frac{?}{7} = \frac{15}{21}$
2. $\frac{2}{3} = \frac{?}{24}$
3. $\frac{1}{9} = \frac{?}{54}$
4. $\frac{?}{5} = \frac{32}{40}$
5. $\frac{1}{?} = \frac{3}{18}$
6. $\frac{2}{13} = \frac{4}{?}$
7. $\frac{5}{8} = \frac{?}{40}$
8. $\frac{-4}{11} = \frac{?}{44}$
9. $\frac{1}{3} = \frac{?}{27}$

10. $\frac{3}{?} = \frac{21}{28}$

11. $\frac{-5}{4} = \frac{50}{?}$

12. $\frac{?}{12} = \frac{30}{72}$

13. $\frac{14}{6} = \frac{42}{?}$

14. $\frac{72}{16} = \frac{144}{?}$

15. $\frac{18}{54} = \frac{?}{27}$

16. $\frac{15}{?} = \frac{3}{75}$

17. $\frac{17}{26} = \frac{?}{78}$

18. $\frac{?}{168} = \frac{6}{21}$

19. $\frac{-36}{?} = \frac{180}{95}$

20. $\frac{?}{-112} = \frac{-54}{28}$

The answers to the previous exercises may be found on page 140.

ADDITION AND SUBTRACTION

Now that we have had an opportunity to work with the basic properties of fractions we will turn our attention towards the basic four operations of addition, subtraction, multiplication and division. This section will cover addition and subtraction and the next section will cover multiplication and division.

To add or subtract two fractions requires that both fractions first be written in a form in which they have the same denominator. Then we simply add or subtract the numerators and put that answer over the common denominator.

Examples: $\frac{2}{13} + \frac{5}{13} = \frac{2+5}{13} = \frac{7}{13}$

$\frac{1}{7} - \frac{4}{7} = \frac{1-4}{7} = \frac{-3}{7}$

When the denominators are different we must apply the Fundamental Property of fractions to change the fractions so they have a common denominator. Any common denominator will do, but it's usually simplest to choose the least common multiple of the two denominators. You may wish to go back and review the section on least common multiples. The abbreviation LCD is used for least common denominator, but in this context the LCD is really the LCM of the denominators.

Example: $\frac{5}{6} + \frac{3}{8}$

The least common multiple of 6 and 8 is 24.

$\frac{5}{6} + \frac{3}{8} = \frac{5 \cdot 4}{6 \cdot 4} + \frac{3 \cdot 3}{8 \cdot 3}$

$= \frac{20}{24} + \frac{9}{24}$

$= \frac{20+9}{24}$

$= \frac{29}{24} = 1\frac{5}{24}$

Example: $\frac{5}{24} - \frac{11}{36}$

$24 = 2^3 \cdot 3^1 \qquad 36 = 2^2 \cdot 3^2 \qquad LCD = 2^3 \cdot 3^2 = 8 \cdot 9 = 72$

$\frac{5}{24} - \frac{11}{36} = \frac{5 \cdot 3}{24 \cdot 3} - \frac{11 \cdot 2}{36 \cdot 2}$

$= \frac{15}{72} - \frac{22}{72}$

$= \frac{-7}{72}$

When mixed numbers are involved in addition and subtraction problems we can sometimes work with the whole number parts and fractional parts separately. Subtraction done in this manner may involve the process of borrowing. But the simplest procedure, and the one which must be used in multiplication and division problems, is to first convert any mixed numbers to improper fractions.

Examples: $2\frac{3}{4} + 5\frac{5}{6}$

$$2\frac{3}{4} + 5\frac{5}{6} = 2 + 5 + \frac{3}{4} + \frac{5}{6}$$

$$= 7 + \frac{9}{12} + \frac{10}{12}$$

$$= 7 + \frac{19}{12}$$

$$= 7 + 1 + \frac{7}{12}$$

$$= 8\frac{7}{12}$$

$5\frac{1}{3} - 2\frac{3}{7}$

$$5\frac{1}{3} - 2\frac{3}{7} = \frac{16}{3} - \frac{17}{7}$$

$$= \frac{16 \cdot 7}{3 \cdot 7} - \frac{17 \cdot 3}{7 \cdot 3}$$

$$= \frac{112}{21} - \frac{51}{21}$$

$$= \frac{61}{21}$$

$$= 2\frac{19}{21}$$

$-3\frac{1}{2} + 2\frac{3}{14}$

$$-3\tfrac{1}{2} + 2\tfrac{3}{14} = \frac{-7}{2} + \frac{31}{14}$$

$$= \frac{-7\cdot 7}{2\cdot 7} + \frac{31}{14}$$

$$= \frac{-49}{14} + \frac{31}{14}$$

$$= \frac{-49 + 31}{14}$$

$$= \frac{-18}{14}$$

$$= \frac{-9}{7} = -1\tfrac{2}{7}$$

EXERCISES: Add or subtract the following numbers.
Reduce all fractions which appear in your final answers.

1. $\frac{5}{9} + \frac{2}{9}$
2. $\frac{7}{8} - \frac{3}{8}$
3. $\frac{3}{4} - \frac{5}{6}$
4. $\frac{2}{3} + \frac{5}{7}$
5. $(-8\frac{1}{3}) + 3\frac{3}{4}$
6. $(-4\frac{1}{5}) + (7\frac{2}{3})$
7. $(6\frac{2}{3}) + (-11\frac{4}{5})$
8. $(7\frac{1}{4}) + (-9\frac{1}{8})$
9. $2\frac{3}{14} - 1\frac{5}{21}$
10. $\frac{2}{3} - 2\frac{1}{2}$
11. $\frac{7}{12} - (2\frac{1}{3})$
12. $\frac{3}{7} - \frac{5}{14}$
13. $8 - 6\frac{2}{3}$
14. $2\frac{7}{15} - 4\frac{3}{10}$
15. $\frac{7}{8} - 2\frac{5}{18}$
16. $\frac{1}{6} - \frac{7}{15}$
17. $\frac{5}{9} + \frac{5}{21}$
18. $1\frac{1}{12} - 1\frac{7}{18}$
19. $2\frac{3}{14} - 1\frac{5}{21}$
20. $6\frac{7}{15} + \frac{9}{20}$
21. $\frac{5}{18} - \frac{19}{24}$
22. $(-2\frac{13}{20}) + (-1\frac{17}{30})$
23. $-\frac{1}{2} + \frac{5}{13}$
24. $1\frac{5}{28} - 3\frac{7}{24}$
25. $5\frac{17}{36} - 1\frac{13}{54}$

The answers to the previous exercises may be found on page 140.

MULTIPLICATION AND DIVISION

Multiplication and division of fractions is simpler than addition or subtraction because we don't need to find a least common denominator. However, any numbers written as mixed numbers should first be changed to improper fractions. This means that before we multiply or divide, we should first convert a mixed number like $2\frac{3}{4}$ into its improper fractional form $\frac{11}{4}$.

The rule for multiplication is very straightforward. Multiply the numerators and multiply the denominators.

Example: $\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$

After multiplying it may be possible to reduce your answer. Reducing a fraction does <u>not</u> mean changing to a mixed number.

Example: $\frac{3}{4} \cdot \frac{10}{9} = \frac{3 \cdot 10}{4 \cdot 9} = \frac{30}{36} = \frac{30 \div 6}{36 \div 6} = \frac{5}{6}$

We should always leave our answers in simplest form and this means all final answers which are fractions should be reduced.

There is a procedure which shows how to reduce before we actually multiply. This procedure is called cancellation. Some teachers avoid the use of the word cancellation because the real operation involved is reduction by division. This is really a special application of the Fundamental Property of fractions. When we cancel, we must perform the same cancellation in both numerator and denominator. The following example shows proper use of cancellation.

Example: $\frac{5}{7} \cdot \frac{2}{5} = \frac{\cancel{5} \cdot 2}{7 \cdot \cancel{5}} = \frac{2}{7}$

The above example shows cancelling 5's from both numerator and denominator. The real operation is division by 5 in both the numerator and denominator. Since $5 \div 5 = 1$, the cancelled numbers should be replaced by 1, but usually this isn't done when other numbers are left in the numerator and denominator. The real danger here is that we might think the cancelled numbers leave nothing, and many students make the mistake of associating the number 0 with nothing. Never insert 0 as a number because of cancellation. Dividing the same number by itself leaves 1.

Example: $\frac{2}{3} \cdot \frac{3}{2} = \frac{2 \cdot 3}{3 \cdot 2} = \frac{\cancel{2} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{2}} = \frac{1 \cdot 1}{1 \cdot 1} = \frac{1}{1} = 1$

Cancellation doesn't always involve identical numbers. Just keep in mind that the true operation is division. The next example shows cancellation by dividing by 2. Since $4 \div 2 = 2$, we replace the cancelled 4 by the number 2. Also, $6 \div 2 = 3$; so, we replace the cancelled 6 by the number 3.

Example: $\frac{4}{3} \cdot \frac{7}{6} = \frac{4 \cdot 7}{3 \cdot 6} = \frac{\overset{2}{\cancel{4}} \cdot 7}{3 \cdot \underset{3}{\cancel{6}}} = \frac{14}{9}$

Multiplication may involve more than one fraction at a time and cancellation may involve several pairs of numbers. But each time cancellation is performed the real operation is division using a number in the numerator together with a number in the denominator.

Example: $\frac{10}{7} \cdot \frac{12}{15} \cdot \frac{7}{9} = \frac{\overset{2}{\cancel{10}} \cdot \overset{4}{\cancel{12}} \cdot \cancel{7}}{\cancel{7} \cdot \underset{3}{\cancel{15}} \cdot \underset{3}{\cancel{9}}} = \frac{8}{9}$

As previously mentioned, when multiplication involves mixed numbers the mixed numbers should first be transformed into improper fractions. Then the multiplication operation may be carried out by multiplying numerators and denominators.

Example: $(3\frac{1}{2})^3 = (\frac{7}{2})^3 = \frac{7}{2} \cdot \frac{7}{2} \cdot \frac{7}{2} = \frac{343}{8}$

Once you've mastered multiplication with fractions there is only one additional rule to know to do division. The rule for dividing fractions is to invert the divisor and then multiply. The divisor is the second fraction. Never invert the first fraction. The next example shows the "invert and multiply" process.

Example: $\frac{3}{4} \div \frac{15}{12} = \frac{3}{4} \cdot \frac{12}{15} = \frac{3 \cdot 12}{4 \cdot 15} = \frac{\overset{1}{\cancel{3}} \cdot \overset{3}{\cancel{12}}}{\cancel{4} \cdot \underset{5}{\cancel{15}}} = \frac{3}{5}$
$\quad\quad\quad

5. $\frac{45}{32} \div \frac{2}{3}$

6. $(-\frac{9}{2}) \div \frac{16}{2}$

7. $(-5\frac{2}{3}) \times (2\frac{1}{4})$

8. $(\frac{5}{6}) \times (3\frac{3}{10})$

9. $(-9\frac{3}{4}) \times (-2\frac{6}{26})$

10. $(5\frac{1}{4}) \times (-11\frac{1}{3})$

11. $(\frac{3}{4}) \div (\frac{1}{2})$

12. $(\frac{13}{40} \div \frac{26}{35})$

13. $(2\frac{1}{17}) \div (1\frac{1}{4})$

14. $(\frac{29}{50}) \div (3\frac{1}{10})$

15. $(-4\frac{5}{8}) \div \frac{1}{4}$

16. $(-5\frac{1}{6}) \div (3\frac{1}{4})$

17. $\frac{1}{5} \div 2 + \frac{3}{5} \cdot \frac{1}{6}$

18. $\frac{1}{2} \div \frac{1}{2} + 1 - \frac{2}{3} \cdot 3$

19. $\frac{3}{4} + (4\frac{1}{2}) \cdot (\frac{1}{3}) \div (\frac{5}{6})$

20. $(3\frac{1}{2}) \cdot (5\frac{1}{3}) + \frac{5}{12} \div \frac{15}{16}$

21. $\frac{3}{10} - \frac{5}{6} \div \frac{1}{4} \cdot \frac{1}{8} + \frac{7}{60}$

22. $\frac{1}{38} + (1\frac{2}{19}) \cdot (1\frac{1}{6}) \div (\frac{7}{8})$

23. $(-\frac{1}{2}) \div (-\frac{7}{8}) + (-\frac{1}{7}) \cdot (-\frac{2}{3})$

24. $-(\frac{2}{15}) \cdot (\frac{1}{4}) \div (\frac{3}{5}) + (-\frac{1}{25})$

25. $(1\frac{1}{6}) \cdot (1\frac{2}{19}) \div (\frac{7}{8}) - \frac{1}{38}$

The answers to the previous exercises may be found on page 140.

COMPLEX FRACTIONS

In addition to the symbol ÷ we can also use the fraction bar to indicate division. Thus, an equivalent way of writing $\frac{2}{3} \div \frac{4}{5}$ is to write:

$$\frac{2}{3} \div \frac{4}{5} = \frac{\frac{2}{3}}{\frac{4}{5}}$$

This last expression is called a complex fraction. The longer fraction bar is the dividing line between the numerator and denominator. In a simple fraction the numerator and denominator are single whole numbers. In a complex fraction either a numerator or denominator or both contain fractions. The numerator of the above complex fraction is the fraction $\frac{2}{3}$. The denominator of the above complex fraction is the fraction $\frac{4}{5}$.

To simplify a complex fraction such as the one above means to transform it into a simple fraction. To simplify the above complex fraction we invert the bottom fraction and multiply with the top fraction.

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$$

This is just our ordinary rule for dividing by a fraction.

Another example of a complex fraction is:

$$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{5}{6}}.$$

This time we cannot simply invert the denominator because the denominator is not a single fraction, it is a sum of two fractions. There are two different methods which can be used to transform this last example into a simple fraction.

The first method is to perform the addition operations in the numerator and denominator. The results of these operations will leave simple fractions in the numerator and denominator. Then, as in the first example, we invert the single fraction in the denominator and multiply with the fraction in the numerator.

$$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{5}{6}} = \frac{\frac{3}{6} + \frac{4}{6}}{\frac{9}{12} + \frac{10}{12}} = \frac{\frac{7}{6}}{\frac{19}{12}} = \frac{7}{6} \cdot \frac{12}{19} = \frac{7 \cdot \cancel{12}^{2}}{\cancel{6} \cdot 19} = \frac{14}{19}$$

A second method is sometimes faster and uses the distributive law and the Fundamental Property of fractions. You may recall from a previous section that the distributive law says that if we multiply a sum or difference of numbers by a number we can turn the problem into

a sum or difference of products.

Thus, $6 \cdot (4 + 5 + 9) = 6 \cdot 4 + 6 \cdot 5 + 6 \cdot 9$

The effect is to distribute the number outside the parentheses across each number inside the parentheses. So, instead of multiplying 6 times the sum, we multiply 6 times each number in the sum, and then we add the products.

To apply the second method to the previous example we first enclose the numerator and denominator of the complex fraction within parentheses.

$$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{5}{6}} = \frac{(\frac{1}{2} + \frac{2}{3})}{(\frac{3}{4} + \frac{5}{6})}$$

Then we determine the least common denominator (LCD) from the denominators of the four fractions which make up the complex fraction. In this example 12 is the least common multiple of 2,3,4, and 6. Now we apply the Fundamental Property of fractions and multiply across the complex fraction with the LCD. Applying the distributive law twice and then adding the results gives a quick answer.

$$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{5}{6}} = \frac{12 \cdot (\frac{1}{2} + \frac{2}{3})}{12 \cdot (\frac{3}{4} + \frac{5}{6})} = \frac{12 \cdot \frac{1}{2} + 12 \cdot \frac{2}{3}}{12 \cdot \frac{3}{4} + 12 \cdot \frac{5}{6}} = \frac{6 + 8}{9 + 10} = \frac{14}{19}$$

When a complex fraction contains mixed numbers we can change the mixed numbers to improper fractions and then apply one of the two given techniques. In the next example we will use the second method of multiplying thru by the LCD. Note that there are only three fractions which make up this complex fraction. The LCD is 30.

Example: Express $\dfrac{-3\frac{1}{2}}{4\frac{2}{3} - 6\frac{3}{5}}$ as a simple fraction.

$$\frac{-3\frac{1}{2}}{4\frac{2}{3} - 6\frac{3}{5}} = \frac{(-\frac{7}{2})}{(\frac{14}{3} - \frac{33}{5})} = \frac{30\ (-\frac{7}{2})}{30\ (\frac{14}{3} - \frac{33}{5})} = \frac{30\ (-\frac{7}{2})}{30 \cdot \frac{14}{3} - 30 \cdot \frac{33}{5}}$$

$$= \frac{-(15 \cdot 7)}{140 - 198} = \frac{-105}{-58} = \frac{105}{58}$$

EXERCISES: Express each of the following complex fractions as simple reduced fractions.

1. $\dfrac{\frac{2}{3}}{\frac{3}{4}}$

2. $\dfrac{2\frac{1}{3}}{\frac{1}{4} + \frac{1}{3}}$

3. $\dfrac{\frac{1}{3} - \frac{6}{5}}{\frac{1}{2} + \frac{1}{5}}$

4. $\dfrac{\frac{2}{3} + \frac{1}{5}}{4\frac{1}{2}}$

5. $\dfrac{-7\frac{2}{5}}{-2\frac{1}{15}}$

6. $\dfrac{\frac{3}{5} + \frac{2}{3}}{\frac{5}{6} - \frac{2}{5}}$

7. $\dfrac{\frac{3}{4} + \frac{2}{3}}{\frac{51}{6}}$

8. $\dfrac{\frac{11}{8}}{1\frac{2}{7} - 6}$

9. $\dfrac{\frac{1}{6} - \frac{2}{3}}{\frac{17}{8} - \frac{1}{4}}$

10. $\dfrac{\frac{3}{20} + \frac{1}{30}}{\frac{2}{3} - \frac{11}{12}}$

11. $\dfrac{\frac{39}{10}}{\frac{2}{5} + \frac{1}{4}}$

12. $\dfrac{\frac{7}{4} - \frac{5}{9}}{\frac{11}{18} + \frac{17}{36}}$

13. $\dfrac{\frac{17}{4}}{\frac{1}{2} - \frac{3}{8}}$

14. $\dfrac{\frac{7}{100} - \frac{28}{50}}{\frac{189}{25}}$

15. $\dfrac{\frac{2}{3} - \frac{5}{6}}{\frac{5}{8} - \frac{1}{16}}$

16. $\dfrac{\frac{4}{15} + \frac{6}{25}}{\frac{3}{5} + \frac{3}{15}}$

17. $\dfrac{2\frac{4}{9} + 1\frac{1}{18}}{1\frac{2}{9} - \frac{1}{6}}$

18. $\dfrac{2\frac{1}{2} - 7\frac{1}{10}}{3\frac{5}{8} + 2\frac{1}{2}}$

19. $\dfrac{5\frac{1}{10} + 4\frac{1}{2}}{\frac{3}{4} + \frac{7}{10} + \frac{1}{5}}$

20. $\dfrac{2\frac{1}{3} - 4\frac{5}{9} + \frac{5}{6}}{1\frac{1}{2} + 3\frac{7}{18} - 4\frac{2}{3}}$

The answers to the previous exercises may be found on page 141.

SECTION III REVIEW PROBLEMS

Reduce the fractions in problems 1-10.

1. $\frac{18}{42}$ 2. $\frac{216}{12}$ 3. $\frac{24}{84}$ 4. $\frac{32}{48}$

5. $\frac{366}{1824}$ 6. $\frac{35}{255}$ 7. $\frac{-180}{-25}$ 8. $\frac{256}{3200}$

9. $\frac{2730}{3570}$ 10. $\frac{6825}{25,935}$

Expand each fraction in problems 11-15 to an equivalent fraction with the indicated denominator.

11. $\frac{2}{3}$ denominator = 27

12. $\frac{5}{7}$ denominator = 56

13. $\frac{12}{15}$ denominator = 75

14. $\frac{6}{9}$ denominator = 126

15. $\frac{11}{17}$ denominator = 187

Tell whether or not the fractions in problems 16-20 are equal.

16. $\frac{1}{13} \stackrel{?}{=} \frac{3}{40}$ 17. $\frac{22}{5} \stackrel{?}{=} \frac{9}{2}$ 18. $\frac{-6}{38} \stackrel{?}{=} \frac{15}{-95}$

19. $\frac{27}{10} \stackrel{?}{=} \frac{8}{3}$ 20. $\frac{52}{80} \stackrel{?}{=} \frac{-78}{120}$

Find the missing numbers in the proportions in problems 21-30.

21. $\frac{14}{10} = \frac{?}{50}$ 22. $\frac{2}{3} = \frac{?}{15}$ 23. $\frac{?}{15} = \frac{40}{75}$

24. $\frac{96}{?} = \frac{32}{40}$ 25. $\frac{-12}{68} = \frac{?}{17}$ 26. $\frac{?}{-5} = \frac{14}{35}$

27. $\frac{3}{7} = \frac{33}{22}$ 28. $\frac{-15}{9} = \frac{-105}{?}$ 29. $\frac{18}{?} = \frac{6}{35}$

30. $\frac{24}{80} = \frac{?}{1040}$

Perform the indicated operations in problems 31-50.

31. $\frac{5}{6} + \frac{3}{8}$ 32. $\frac{1}{2} - \frac{4}{9}$ 33. $\frac{3}{4} \cdot \frac{18}{21}$

34. $\frac{5}{6} \div \frac{1}{2}$ 35. $\frac{5}{9} - \frac{3}{12}$ 36. $\frac{4}{3} + \frac{1}{8}$

37. $2\frac{3}{4} + 9\frac{1}{6}$ 38. $(-2\frac{1}{2})(3\frac{2}{3})$ 39. $(6\frac{1}{8}) \div (5\frac{3}{4})$

40. $\frac{1}{2} + \frac{1}{6} \cdot \frac{3}{8}$ 41. $\frac{7}{9} - \frac{1}{6} \div \frac{2}{3}$

42. $\frac{7}{12} - (\frac{3}{4} - \frac{1}{3})$ 43. $\frac{4}{25} \div \frac{3}{5} \cdot \frac{1}{2}$

44. $(2\frac{2}{3})^2$ 45. $8 - 6\frac{2}{3} + \frac{1}{3}$

46. $\frac{5}{8} \div (2\frac{5}{8}) - \frac{3}{8}$ 47. $\frac{-1}{3} \div 4 + \frac{1}{6} \div \frac{1}{8}$

48. $\frac{3}{4} + (3\frac{1}{6})(\frac{4}{9}) + \frac{-5}{8}$ 49. $(6\frac{2}{7}) \div (2\frac{5}{14}) - \frac{3}{8} + \frac{1}{5}$

50. $\frac{6}{7} + \frac{2}{3} - \frac{3}{7} \cdot \frac{1}{2} \div (1\frac{1}{2})$

Express each of the following complex fractions as simple reduced fractions.

51. $\dfrac{\frac{1}{3}}{\frac{1}{2}}$ 52. $\dfrac{\frac{2}{3} + \frac{1}{6}}{\frac{5}{12} - \frac{3}{4}}$ 53. $\dfrac{2\frac{2}{3}}{\frac{5}{6} - \frac{7}{9}}$

54. $\dfrac{\frac{5}{8} - \frac{3}{2}}{(4\frac{2}{3})}$

55. $\dfrac{\frac{1}{4} + \frac{2}{5}}{\frac{3}{10} - (4\frac{1}{5})}$

56. $\dfrac{(2\frac{1}{3}) + (1\frac{5}{6})}{(1\frac{3}{4}) - (3\frac{7}{12})}$

The answers to the Review Problems may be found on page 142.

SECTION IV DECIMALS

DECIMAL NOTATION ADDITION AND SUBTRACTION

There are several ways in which we can represent the numbers in our number system. With the advent of pocket calculators and digital watches the use of decimals is becoming even more popular, although decimal numbers have always been used in scientific work.

The basis for decimal number notation is an extension of the base ten place-value system we use for writing whole numbers. The following diagram shows the place values of the digits of the whole number 5,963,195 which is read as "five million sixty-three thousand one hundred ninety five."

millions	hundred thousand's	ten thousand's	thousand's	hundred's	ten's	units or one's
5,	9	6	3,	1	9	5

Each place value is ten times the place value immediately to the right.

To write a number in decimal form we use a decimal point "." and continue writing digits to the right of the decimal point. The first place to the right of the decimal point is the "tenths" place and each place value is divided by ten to obtain the next place value to the right.

The following diagram shows the place values of the first few decimal places to the right of the decimal point.

.	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths	ten millionths
decimal point							

When reading decimal numbers we should avoid the use of the word "point". In fact, the decimal point should be read as "and" followed by the place value of the right-most decimal digit. Thus, "5.23" should not be read as "five point two three", but rather should be read as "five and twenty three hundredths". The following examples show how to read various decimals.

6.3	Six and three tenths
0.15	Zero and fifteen hundreths
2.08	Two and eight hundredths
3.456	Three and four hundred fifty six thousandths
7.032	Seven and thirty two thousandths
4.0005	Four and five ten thousandths
1.7689	One and seven thousand six hundred eighty nine ten thousandths

It is important to be able to read decimals using the place value of the digits and avoid saying "point". Reading a decimal in this manner provides a way to convert decimals into fractions.

Examples: Convert 2.6 to a reduced fraction. 2.6 should be read as two and six tenths.

$$\text{Thus, } 2.6 = 2\frac{6}{10} = \frac{26}{10} = \frac{26\div 2}{10\div 2} = \frac{13}{5}$$

Convert 3.25 to a reduced fraction; 3.25 should be read as three and twenty-five hundreths.

$$\text{Thus, } 3.25 = 3\frac{25}{100} = \frac{325\div 25}{100\div 25} = \frac{13}{4}$$

Convert -6.024 to a reduced fraction. -6.024 should be read as negative six and twenty four thousands.

$$\text{Thus, } -6.024 = -6\frac{24}{1000} = -6\frac{12}{500} = -6\frac{6}{250} = -6\frac{3}{125}$$

$$= -\frac{753}{125}.$$

The last example shows we can reduce the fractional part before converting the mixed number to an improper fraction.

The procedure to write any whole number as a fraction is to simply put the number over 1. Thus, 3 written as a fraction becomes $\frac{3}{1}$.

The procedure to write a whole number as a decimal is just as simple. All we need to do is to place a decimal point to the right of the number. Thus, the whole number 3 when written as a decimal becomes 3. and 18 written as a decimal is 18..

The next three examples show that any zeros to the right of the decimal point but to the left of other digits are important because they determine different numbers.

Examples: $2.05 = 2\frac{5}{100} = 2\frac{1}{20}$

$2.005 = 2\frac{5}{1000} = 2\frac{1}{200}$

$2.0005 = 2\frac{5}{10000} = 2\frac{1}{2000}$

All three decimals are different.

However, the next examples show that we can insert or delete any zeros that appear as the rightmost digits. Depending on the problem we are working, there are times when we will need to insert or delete zeros as the rightmost digits in a decimal.

Examples: $3.6 = 3\frac{6}{10} = 3\frac{3}{5}$

$3.60 = 3\frac{60}{100} = 3\frac{30}{50} = 3\frac{3}{5}$

$3.600 = 3\frac{600}{1000} = 3\frac{300}{500} = 3\frac{3}{5}$

$3.6000 = 3\frac{6000}{10000} = 3\frac{3000}{5000} = 3\frac{3}{5}$

Thus, all four decimals are equal.

$3.6 = 3.60 = 3.600 = 3.6000$

The previous examples have shown that every decimal number can be made into a fraction. And the rules for adding or subtracting fractions first require finding a common denominator. Adding or subtracting decimals requires the same, but the process in terms of decimals amounts to vertically lining up the decimal points.

Example: Add $3.6 + 2.551$ First line up the decimal points vertically.

$$\begin{array}{r} 2.551 \\ 3.6 \\ \hline \end{array}$$

Although it isn't mandatory in addition, we can insert trailing zeros to make both numbers have the same number of digits to the right of the decimal points. So we could also have arranged our work as:

$$\begin{array}{r} 2.551 \\ 3.600 \\ \hline \end{array}$$

Then we simply add the numbers as if they were whole numbers and place the decimal point in our answer directly beneath the two decimal points of the numbers we are adding or subtracting.

$$\begin{array}{r} 2.551 \\ +\ 3.600 \\ \hline 6.151 \end{array}$$

The rules for operating with positive and negative decimals are exactly the same rules we use for operating with whole numbers. When subtracting decimals it is necessary to add trailing zeros to the top number to make the same number of decimal places as in the bottom number.

Thus, change $\begin{array}{r} 42.6 \\ -\ 3.794 \\ \hline \end{array}$ to $\begin{array}{r} 42.600 \\ -\ 3.794 \\ \hline \end{array}$

and then subtract as if the numbers were whole numbers. The rule for decimal point placement in subtraction is the same as the rule for addition.

$$42.6 - 3.794 = 38.806$$

Example: Subtract $^{-}3.7 - (^{-}1.129)$

We change the sign of the number being subtracted and change the subtraction operation to addition. $^{-}3.7 - (^{-}1.129) = {}^{-}3.7 + (^{+}1.129)$. Now we are adding a negative and a positive. The sign of our answer is the sign of the larger number, in this case negative because 3.7 is larger than 1.129. But the numerical value of the addition is the difference between 3.7 and 1.129. To find this difference we may write:

$$\begin{array}{r} 3.700 \\ -1.129 \\ \hline 2.571 \end{array}$$

The final result is that $^{-}3.7 - (^{-}1.129) = -2.571$

EXERCISES: Convert the decimals in problems 1-10 to reduced fractions. Perform addition and subtraction in problems 11-30.

1. 4.3	2. 1.26	3. -2.22
4. 3.375	5. -4.50	6. 5.875
7. -6.33	8. 3.625	9. -10.65

10. 4.888

11. 5.4 + 12.32

12. 219.716 + 3.51

13. 712.4 - 2.163

14. 88. + 0.362

15. 21 - 4.76

16. 36.32 - 41.690

17. 5.71 - (⁻12.348)

18. ⁻23.7 + ⁻4.76

19. ⁻0.001 + ⁻3.027

20. 4.1909 - 8.0908

21. 5.4 + ⁻0.075 - 10

22. 2.5 + 0.25 + 0.025 + 0.0025

23. 63.36 - 36.63 + 66.33 - 33.66 + 36 - 63

24. 9.67 - (8.35 - ⁻2.1 + 10.34)

25. (0.54 - 6.88) - (2.54 - 7.936)

26. 20.5 - 25.6094 + ⁻1.632

27. ⁻5.846 + ((2.148 + 3.49) - (8.32 - 0.034))

28. ⁻4.27 - (((13.31 + ⁻4.27) - (4.19 - 91.8)) + 58.9)

29. (0.667 + ⁻4.3 + ⁻3.79) - ((88.77 - 77.88) + ⁻0.1234)

30. (37.42 - (5.66 - (2.11 - (4.61 - 3.5))))

The answers to the previous exercises may be found on page 143.

DECIMAL MULTIPLICATION AND DIVISION

Multiplication and division of decimals also requires proper decimal point location in the answer. Multiplication is performed as in the case of whole numbers, but then we add the number of decimals to find the total number of places to be counted in the answer. We begin counting decimal places in our answer with the rightmost digit in the answer.

Example:

```
  3.26          2 decimal places
x 7.5         + 1 decimal place
 1630           3 decimal places total to be
2282              counted in the answer
24450  =  24.450
```

The answer to the above example is a little more than 24 and this answer seems reasonable since if we simply dropped the decimals in 3.26 and 7.5 we would have: $3 \cdot 7 = 21$.

The rule for decimal point location in the answer to a division problem is closely related to the fundamental property of fractions. Before we make this connection however, we will first look at an example of decimal division. The rule states that when the divisor is a whole number the decimal point in the answer goes directly above the decimal point in the dividend.

Example: $43.68 \div 16$

```
    2.73
16.|43.68
   32
   11 6
   11 2
      48
      48
```

Because the divisor is a whole number the decimal point in the answer is directly above the decimal point in the dividend. No decimal points are shifted.

If the divisor is not a whole number then we must first shift the decimal point to the right enough places to make the divisor into a whole number. But then we also shift the decimal point in the dividend the same number of places as determined by the divisor.

Example:

```
        195  =  1.95
2.36|4.6020
    2 36
    2 242
    2 124
      1180
      1180
```

First shift decimal in divisor 2 places to the right to make the divisor a whole number. Then shift decimal in the dividend the same number of places as in the divisor. The decimal point in the answer is directly above the shifted decimal point in the dividend.

We must remember to only try and make the divisor into a whole number, not the dividend. If any decimals are shifted, the decimal in the divisor is shifted first. When the decimal in the dividend is shifted, it is shifted the same number of places as the decimal in the divisor was shifted.

One shortcut for multiplying a decimal by 10 or 100 or 1000 or any power of 10 will help explain the decimal shifting in division. If a decimal is multiplied by a power of 10, shift the decimal point to the right the number of places there are zeros in the power of 10.

Examples: $(1.239) \cdot (100) = 123.9$ Shift decimal right 2 places because there are 2 zeros in 100.

$(56.8914) \cdot (1000) = 56891.4$ Shift decimal right 3 places because there are 3 zeros in 1000.

Now if we go back and review the Fundamental Property of fractions we can better understand the decimal shifting process in division. The previous example of division will do. Remember that the guiding principle is to make the divisor into a whole number.

The three ways of writing a division problem are:

$4.6020 \div 2.36$ $\qquad$ $2.36\overline{)4.6020}$ $\qquad$ $\dfrac{4.6020}{2.36}$

Let's take the fractional form: $\dfrac{4.6020}{2.36}$. In order to make the divisor 2.36 into a whole number, we multiply it by 100. But, the Fundamental Property of Fractions states that if we multiply the denominator by 100 we must do the same to the numerator. This is why the decimal in the divisor shifts first and why the decimal in the dividend shifts the same number of places as the decimal in the divisor.

$$\frac{4.6020}{2.36} = \frac{(4.6020)\cdot 100}{(2.36)\cdot 100} = \frac{460.20}{236.} = 236.\overline{)460.20}$$

Division with decimals provides a direct way of converting fractions to decimals. All we do is divide the denominator into the numerator using decimals. In the process of dividing it may be necessary to add 0's to the dividend so the division process may be continued.

Example: Convert $\frac{3}{8}$ to a decimal.

$$
\begin{array}{r}
.375 \\
8.\overline{)3.000} \\
24 \\
\hline
60 \\
56 \\
\hline
40 \\
40 \\
\hline
\end{array}
$$

← add 0's to dividend to continue division.

When performing decimal division the question arises as to how many decimal places should be carried in the answer. Our general rule of thumb is to carry the division out to four decimal places and then round the final answer back to three places. Rounding to the third place means add 1 to the third decimal digit if the fourth decimal digit is 5 or more. Otherwise, just drop any digits past the third decimal place.

Example: Convert $\frac{6}{13}$ to a decimal.

$$
\begin{array}{r}
.4615 \\
13.\overline{)6.0000} \\
52 \\
\hline
80 \\
78 \\
\hline
20 \\
13 \\
\hline
70 \\
65 \\
\hline
\end{array}
$$

carry answer to 4 decimals before rounding
add 0's to continue division

Answer: Rounded to three places $\frac{6}{13} = .462$

EXERCISES: Perform the operations in problems 1-20. Convert the fractions in problems 21-25 to decimals.

1. $(-6.23)(5.7)$
2. $(12.87) \div (^{-}4.5)$
3. $(^{-}33.488) \div (3.2)$
4. $(19.79) \div (0.01)$
5. $(^{-}13.261) \div (0.0001)$
6. $(2.6)(1.2)(3.5)$
7. $(^{-}2.14)^2$
8. $(^{-}3.2)^3$
9. $(-52.3)(^{-}1.56)$
10. $(.0028)(17.9)$
11. $4.02 - (2.8)(3.1)$
12. $(^{-}2.6)(9.32 + (^{-}13))$
13. $[(5.8)(^{-}3.6) - 12] - [8 + (^{-}6.5)(4)]$
14. $[(^{-}8.7)^2 + (^{-}.1)^2] + [^{-}8.7 + ^{-}0.1]^2$

15. $[5.2 + (^-72) \div (1.8)(^-3)] - [15 + (^-21.3)]$

16. $[8.8 + (^-5)^2] \cdot [^-4 + (^-2.9)(^-3.6)]$

17. $[(^-1.4)^2 - (3.5)^2]^2$

18. $|^-4.36|(^-2.63)$

19. $(2.2)(11.7 - 13.9) \div (^-4)^2$

20. $[4.8 - 7.3]^2 - [6.9-3.2]^2 \ (4.5) \div (^-5)$

21. $\frac{5}{8} =$

22. $\frac{50}{16} =$

23. $\frac{13}{40} =$

24. $\frac{117}{25} =$

25. $\frac{239}{75} =$

The answers to the previous exercises may be found on page 143.

SQUARE ROOTS

A number which is the square of another number is called a perfect square. The following is a list of integers and perfect squares of those integers.

1	2	3	4	5	6	7	8	9	10	11	12
1	4	9	16	25	36	49	64	81	100	121	144

The square root of a given number is the number whose square equals the given number. $\sqrt{\ }$ is the symbol for square root.

Examples:

$\sqrt{25} = 5$ because $5^2 = 25$

$\sqrt{64} = 8$ because $8^2 = 64$

$\sqrt{289} = 17$ because $17^2 = 289$

$\sqrt{5\frac{4}{9}} = \frac{7}{3}$ because $(\frac{7}{3})^2 = 5\frac{4}{9}$

$\sqrt{20.25} = 4.5$ because $(4.5)^2 = 20.25$

Not every number has a square root. For example, $\sqrt{-16}$ does not exist. While it might be tempting to say the answer is negative 4, the rules for multiplying signed numbers prevent us from finding $\sqrt{-16}$ or the square root of any other negative number.

NEGATIVE NUMBERS DO NOT HAVE SQUARE ROOTS

Trying to find the square root of a negative number is an illegal mathematical operation, just as is trying to divide by zero.

Also, the result of taking the square root of a number can never be a negative answer. Even though $(^-6)^2 = {}^+36$, $\sqrt{36} = {}^+6$ only. The square root of any number is always positive, except for 0. $\sqrt{0} = 0$.

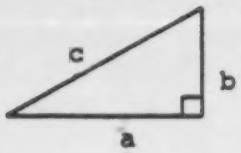
Only perfect squares have square roots which come out even. All other square roots are infinite non-repeating decimals. Such square roots can only be written on paper using a limited number of decimals.

Example: $\sqrt{23} \approx 4.7958$ The symbol $\approx$ means "approximately equals."

$(4.7958)^2 = 22.99969764$ which is almost 23.

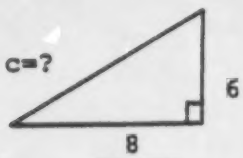
One of the first applications of square roots involves one of the most famous results from geometry called the Pythagorean Theorem. The statement says that the square of the longest side in a right triangle is equal to the sum of the squares of the two shorter sides.

The diagram below illustrates the Pythagorean Theorem.



$$c^2 = a^2 + b^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

Example: Find the length of side c in the right triangle below.



$$c = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$$

There are several methods which can be used to compute square roots, but our purpose here is not computation. An electronic calculator is the perfect instrument for doing numerical computations. But of course a calculator cannot think for you.

We will be more concerned with methods of how to manipulate square roots rather than how to compute square roots. Our purpose is to learn properties of square roots which will be necessary for understanding other processes in algebra.

In Section III we learned how to reduce fractions. When a fraction is reduced the numbers are made smaller and the terms of the fraction are left in their simplest form. In this section we are going to learn the process to reduce square roots.

One of the fundamental properties of square roots is:

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

This property says that the square root of a product is equal to the product of the square roots of the two numbers which originally made the product. This property does <u>not</u> work when the operation is addition or subtraction. A similar property holds when the operation is division.

To reduce the square root of a number, write the number as a product, where the first number in the product is a perfect square. Then apply the above property of square roots as in the examples below.

Example: Reduce $\sqrt{75}$. 25 is a perfect square which divides evenly into 75. $\sqrt{25} = 5$

$$\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5 \cdot \sqrt{3}$$

$5 \cdot \sqrt{3}$ is the reduced form of $\sqrt{75}$.

Example: Reduce $\sqrt{45}$

9 is a perfect square which divides evenly into 45.

$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3 \cdot \sqrt{5}$$

It is important to always write the first number in the product as the perfect square. For example, there are several ways to write 50 as a product. But if we tried to reduce $\sqrt{50}$ we should not write $\sqrt{50} = \sqrt{5 \cdot 10} = \sqrt{5} \cdot \sqrt{10}$ because neither 5 nor 10 is a perfect square. But, if we write $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5 \cdot \sqrt{2}$ then, we have chosen a perfect square (25) as the first number in the product and we can use the fact that $\sqrt{25} = 5$.

It is best to find the largest perfect square which divides evenly into our given number, otherwise we may have to reduce twice as illustrated in the next example.

Example: Reduce $\sqrt{108}$

9 is a perfect square which divides into 108.
$\sqrt{108} = \sqrt{9 \cdot 12} = \sqrt{9} \cdot \sqrt{12} = 3 \cdot \sqrt{12}$ but $\sqrt{12}$ can itself be further reduced so we continue:

$$\sqrt{108} = 3 \cdot \sqrt{12} = 3 \cdot \sqrt{4 \cdot 3} = 3 \cdot \sqrt{4} \cdot \sqrt{3} = 3 \cdot 2 \cdot \sqrt{3} = 6 \cdot \sqrt{3}$$

We could have made a simpler solution by first using 36 as the largest perfect square which divides into 108.

$$\sqrt{108} = \sqrt{36 \cdot 3} = \sqrt{36} \cdot \sqrt{3} = 6 \cdot \sqrt{3}$$

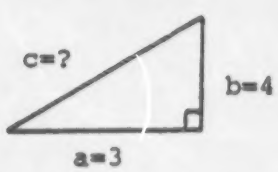
Knowing the list of perfect square integers is a great aid to reducing square roots of large numbers. The last example is below.

Example: Reduce $\sqrt{288}$

$$\sqrt{288} = \sqrt{144 \cdot 2} = \sqrt{144} \cdot \sqrt{2} = 12 \cdot \sqrt{2}$$

EXERCISES: Use the Pythagorean Theorem to find the longest side of the triangles in problems 1-8. Reduce the square roots in problems 9-25.

1.



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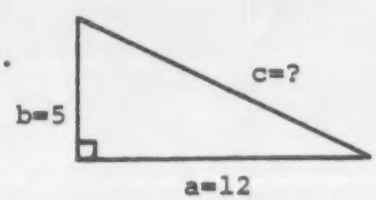

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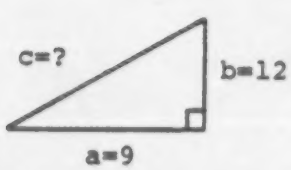

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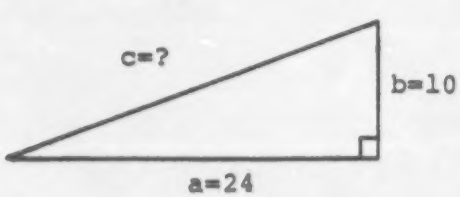

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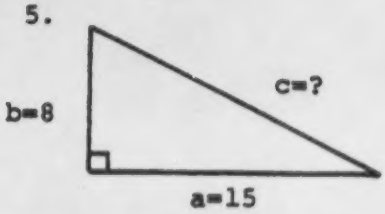

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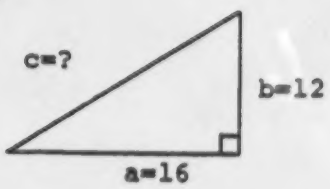

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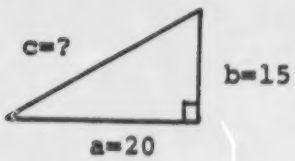

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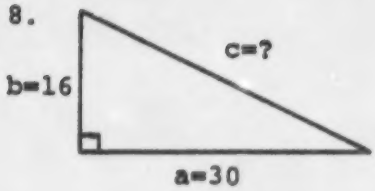


9. $\sqrt{18}$ =

10. $\sqrt{28}$ =

11. $\sqrt{32}$ =

12. $\sqrt{72}$ =

13. $\sqrt{98}$ =

14. $\sqrt{192}$ =

15. $\sqrt{500}$ =

16. $\sqrt{150}$ =

17. $\sqrt{117}$ =

18. $\sqrt{147}$ =

19. $\sqrt{243}$ =

20. $\sqrt{450}$ =

21. $\sqrt{363}$ =

22. $\sqrt{338}$ =

23. $\sqrt{392}$ =

24. $\sqrt{675}$ =

25. $\sqrt{432}$ =

The answers to the previous exercises may be found on page 144.

PERCENT

The three kinds of numbers commonly used in mathematics are fractions, decimals, and percent. We have previously discussed fractions and decimals. This section will briefly describe percents.

A percent is a fraction whose denominator is 100. The symbol for percent is %. Thus, percents are really not a new kind of number, but only another way of writing fractions or decimals.

Examples: $5\% = \frac{5}{100}$ $\quad 30\% = \frac{30}{100}$

$\frac{17}{100} = 17\%$ $\quad 100\% = \frac{100}{100} = 1$

$7\frac{1}{2}\% = \frac{7\frac{1}{2}}{100}$ $\quad 6.75\% = \frac{6.75}{100}$

Our first concern is learning how to convert between decimals and percents. To change a per cent to a decimal, move the decimal point two places to the left and drop the percent symbol.

Examples: $5\% = 5.\% = 0.05$

$8.6\% = 0.086$

$7\frac{1}{2}\% = 7.5\% = 0.075$

$9\ 3/4\% = 9.75\% = 0.0975$

Note that in the previous two examples the percent numbers contained fractions and we first converted those fractions to their decimal equivalents in order to obtain a decimal point. We still retained the % symbol until we shifted the decimal point two places to the left, then and only then did we drop the % symbol.

To convert a decimal number to a percent we apply the opposite of the above rule. In converting a decimal number to a percent, shift the decimal point two places to the right and add a % symbol to the right of the number.

Examples: $0.07 = 7.\%$

$0.025 = 2.5\%$

$2.19 = 219.\%$

$0.005 = 0.5\%$

You should now be able to convert between numbers written in either decimal form or percent form. Sometimes we also need to convert between fractions and percents.

Example: How do you write $\frac{7}{25}$ as a fraction with a denominator of 100?

$$\frac{7}{25} = \frac{?}{100}$$

$$\frac{7}{25} = \frac{7 \cdot 4}{25 \cdot 4} = \frac{28}{100}$$

By definition, a percent is a fraction whose denominator is 100.
So we have: $\frac{7}{25} = \frac{28}{100} = 28\%$

The previous example showed one method for converting a fraction to a percent. In that example, it was convenient to apply the Fundamental Property of fractions and change the fraction $\frac{7}{25}$ to a fraction with a denominator of 100. Many times the number in the denominator is not easily converted to 100. Thus, a more common method of changing a fraction to a per cent is to first convert the fraction to a decimal (by dividing) and then change the decimal to a per cent.

Examples: Convert the following fractions to percents by first changing the fractions to decimals.

$$\frac{13}{40} = 0.325 = 32.5\%$$

$$\frac{7}{8} = 0.875 = 87.5\%$$

$$\frac{29}{20} = 1.45 = 145.\%$$

$$\frac{17}{20} = .85 = 85\%$$

To convert a percent to a fraction we can first convert the percent to a decimal and then convert the decimal to a reduced fraction.

Examples: Change the following percents to reduced fractions.

$$38\% = 38.\% = .38 = \frac{38}{100} = \frac{19}{50}$$

$$75\% = 75.\% = .75 = \frac{75}{100} = \frac{3}{4}$$

$$6.5\% = 0.065 = \frac{65}{1000} = \frac{13}{200}$$

$$9.25\% = 0.0925 = \frac{925}{10,000} = \frac{37}{400}$$

We now summarize the four rules we have developed in this section for working with percents.

1. Changing a percent to a decimal. Move the decimal point two places to the left and drop the percent symbol.
2. Changing a decimal number to a percent. Move the decimal point two places to the right and add a % symbol to the right of the number.
3. Changing a fraction to a percent. First, convert the fraction to a decimal (by dividing) and then use the above rule to convert the resulting decimal to a percent.
4. Changing a percent to a fraction. First, convert the percent to a decimal using the above rule and then convert the decimal to a reduced fraction.

EXERCISES: Convert the percents in problems 1-10 to decimals.

1. 38%
2. 9.5%
3. 3.25%
4. 0.05%
5. 5½%
6. $6\frac{3}{4}\%$
7. 100%
8. $\frac{1}{4}\%$
9. .09%
10. 66.8%

Convert the decimals in problems 11-20 to percents.

11. 0.05
12. 2
13. 0.432
14. 3.3
15. 1.5
16. 0.04
17. 1
18. 0.15
19. 0.333
20. 1.08

Convert the fractions in problems 21-30 to percents.

21. $\frac{3}{4}$
22. $\frac{2}{5}$
23. $\frac{5}{8}$
24. $2\frac{3}{8}$

25. $1\frac{3}{10}$

26. $\frac{12}{25}$

27. $\frac{3}{20}$

28. $2\frac{7}{100}$

29. $\frac{37}{5}$

30. $2\frac{1}{3}$

Convert the percents in problems 31-40 to reduced fractions.

31. 35%

32. 1.5%

33. 137.5%

34. 100%

35. 62.5%

36. 180%

37. 30%

38. 10.8%

39. $2\frac{1}{2}\%$

40. $\frac{1}{4}\%$

The answers to the previous exercises may be found on page 144.

SECTION IV REVIEW PROBLEMS

Convert the decimals in problems 1-10 to reduced fractions.

1. 5.6
2. 0.48
3. -5.946
4. 6.875
5. 0.028
6. 3.66
7. -2.4375
8. 3.85
9. -14.24
10. 6.475

Perform the indicated operations in problems 11-35.

11. $2.41 + 3.007$
12. $4.65 - 8.32$
13. $9 - 10.37$
14. $(3.6)(2.5)$
15. $(-61.5) \div (-7.5)$
16. $(-5.6)(3.002)$
17. $(26.76)\ (-2.23)$
18. $(5.4)^2$
19. $(-5.4)^2$
20. $(2.3)^3$
21. $(-2.3)^3$
22. $(-1.7)^3$
23. $(-4.1)(5.7) + 8$
24. $(8.2) \div (0.04)$
25. $(0.00268) \div (0.08)$
26. $^-5.9 + (6.1)(^-2.4)$
27. $^-14 + (16.59) \div (2.37)$
28. $^-35 + (^-83.6) \div (^-7.6)$
29. $(^-5.5)^2 \div (0.05)$
30. $(^-441.8) \div (^-9.4)^2$
31. $(^-3.2)^2 + (^-2.3)^3$
32. $\left[(-6.7) + (3.8)^2\right]\left[(-504) \div 6.3\right]$
33. $(8.3)(15.6-18.91) \div (33.1) - (6.7-8.56)$
34. $[^-11.8 + (^-1.53) \div (17)(^-4.2)] - (17.32-19.18)$
35. $[12.3 + (17.1) \div (^-1.9)(5)] - [33.37 + (8.6)^-5.1)]$

Reduce the following square roots.

36. $\sqrt{27}$ 37. $\sqrt{80}$ 38. $\sqrt{63}$

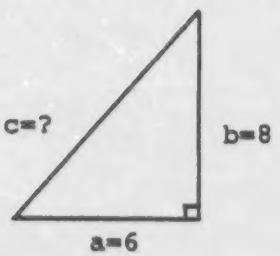
39. $\sqrt{72}$ 40. $\sqrt{28}$ 41. $\sqrt{300}$

42. $\sqrt{54}$ 43. $\sqrt{112}$ 44. $\sqrt{98}$

45. $\sqrt{192}$

Find the length of the longest side of each triangle in problems 46-49.

46.



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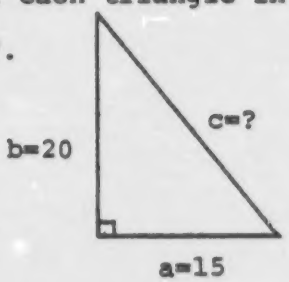

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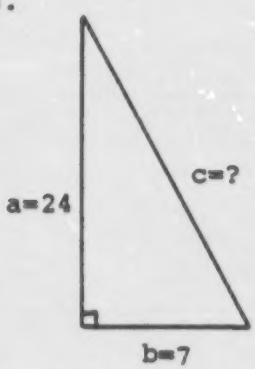

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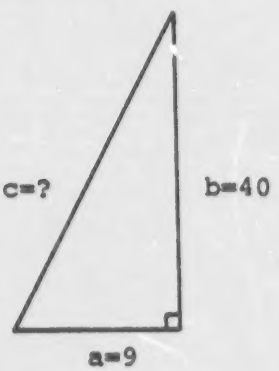


Convert the fractions in problems 50-54 to decimals.

50. $\frac{7}{8}$ 51. $\frac{27}{16}$ 52. $\frac{11}{20}$ 53. $\frac{111}{75}$ 54. $\frac{19}{40}$

Convert the percents in problems 55-59 to decimals.

55. 42% 56. 7.2% 57. 0.01% 58. 4.75% 59. $7\frac{3}{4}\%$

Convert the decimals in problems 60-64 to percents.

60. 0.08 61. 0.093 62. 0.567 63. 1.21 64. 0.0041

Convert the fractions in problems 65-69 to percents.

65. $\frac{1}{2}$ 66. $\frac{1}{4}$ 67. $\frac{3}{5}$ 68. $3\frac{7}{8}$ 69. $\frac{28}{35}$

Convert the percents in problems 70-75 to reduced fractions.

70. 44% 71. 37.5% 72. 60% 73. 12.5% 74. 1.8%

75. 175%

The answers to the Review Problems may be found on page 145.

SECTION V EXPONENTS

ZERO AND NEGATIVE EXPONENTS

When we previously defined exponents we could have used any number as the base (bottom number) but we required the exponent to be a positive whole number. The exponent was used to indicate how many times to repeat the base as a factor in a product. Thus,

$7^5 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ (five 7's multiplied)

We will now show how it is possible to use any integer as an exponent. Consider the following list of powers. Notice that the exponents decrease by one as you read down the list of exponents on the left. Note also how each number in the list on the right equals the previous number divided by 2.

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = ?$

If we simply continue the pattern of dividing the previous number by 2 then the last number in the right column should be $2 \div 2 = 1$. Using this scheme it seems natural to define $2^0 = 1$.

If we make a similar list of powers using 3 as a base we discover a pattern in which each number in the right column equals the number above it divided by 3.

$3^4 = 81$

$3^3 = 27$

$3^2 = 9$

$3^1 = 3$

$3^0 = ?$

Continuing the pattern of dividing each previous number by 3 should make it seem natural to define $3^0 = 3 \div 3 = 1$. In fact, we can do this for any base except 0 and we now make the following definition:

If $x \neq 0$ we define $x^0 = 1$.

Thus, any number to the power of 0 equals 1 except that 0^0 is not defined.

By continuing the patterns of dividing by 2 or 3 each time the exponent is decreased by 1 we obtain the following lists.

$$2^2 = 4 \qquad 3^2 = 9$$
$$2^1 = 2 \qquad 3^1 = 3$$
$$2^0 = 1 \qquad 3^0 = 1$$
$$2^{-1} = \frac{1}{2} \qquad 3^{-1} = \frac{1}{3}$$
$$2^{-2} = \frac{1}{4} \qquad 3^{-2} = \frac{1}{9}$$
$$2^{-3} = \frac{1}{8} \qquad 3^{-3} = \frac{1}{27}$$
$$2^{-4} = \frac{1}{16} \qquad 3^{-4} = \frac{1}{81}$$

In general then we can make the following definition for negative exponents.

> If n is a positive integer and x is any nonzero number then we define:
>
> $$x^{-n} = \frac{1}{x^n}$$

Of course, it doesn't make any sense to say that the base is repeated a negative number of times. But the above definition is consistent with the way in which exponents are used. After gaining a little more experience with negative and zero exponents we will develop the standard rules used with exponents. At that time we can again justify the reasons for defining zero and negative exponents.

The concept of the reciprocal of a number may help with understanding how to do computations with negative exponents. If x is any nonzero number, then $\frac{1}{x}$ is called the reciprocal of x. If x is a fraction of the form $x = \frac{m}{n}$, then:

$$\frac{1}{x} = \frac{1}{\frac{m}{n}} = 1 \cdot \frac{n}{m} = \frac{n}{m}$$

In other words, the reciprocal of a fraction is the fraction turned upside down. We have already used this reciprocal idea in the work in dividing by fractions. The rule for dividing by a fraction, "invert

and multiply", may be stated as "multiply by the reciprocal of the divisor."

Our definition for a negative exponent then says that x to a negative power equals the reciprocal of x to the exponent without the negative sign.

Examples: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$$(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27}$$

$$(\tfrac{3}{4})^{-2} = \frac{1}{(\frac{3}{4})^2} = \frac{1}{\frac{9}{16}} = \frac{16}{9}$$

Sometimes students think the answer to 2^{-3} should somehow be negative, perhaps because the exponent is negative and they know exponents involve multiplication. Such reasoning is incorrect. What is being multiplied is the base, and if the base is positive, then the final answer can never be negative, regardless of what the exponent is. We never multiply with the exponent.

Examples: $(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$

$$(-\tfrac{1}{2})^{-4} = \frac{1}{(-\frac{1}{2})^4} = \frac{1}{(\frac{1}{16})} = 16$$

$$(-3\tfrac{1}{2})^{-3} = \frac{1}{(-3\frac{1}{2})^3} = \frac{1}{(-\frac{7}{2})^3} = \frac{1}{-\frac{343}{8}} = \frac{-8}{343}$$

As you can probably see in the above examples, the first steps in simplifying a problem with a negative exponent are to drop the negative sign and write the reciprocal of the same base to the same power without the negative sign.

Exercises: Simplify problems 1-15 by writing the answers without negative exponents. In problems 16-30 first simplify all negative exponents and then perform the indicated operations.

1. $4^{-2} =$
2. $2^{-4} =$
3. $(-1)^{-5} =$
4. $(-3)^{-2} =$
5. $5^{-3} =$
6. $(-10)^{-3} =$
7. $(\frac{1}{4})^{-2} =$
8. $(\frac{1}{8})^{-2} =$
9. $(\frac{2}{3})^{-1} =$
10. $(-\frac{2}{3})^{-1} =$
11. $(-\frac{3}{4})^{-3} =$
12. $(5\frac{1}{2})^{-2} =$
13. $(-1)^{0} =$
14. $(5689)^{0} =$
15. $(-\frac{3}{7})^{-2} =$
16. $(-2)^{-2} + (-3)^{-2} =$
17. $(-2 + -3)^{-2} =$
18. $[(2)^{-2}][(-3)^{-3}]$
19. $2(5)^{2} + (\frac{1}{2})^{-3} =$
20. $[(\frac{3}{4})^{-1}] \div [(\frac{2}{3})^{-3}]$
21. $(\frac{4}{7})^{2} - (\frac{7}{4})^{-2} =$
22. $(\frac{3}{5})^{-2} + (\frac{4}{3})^{2} =$
23. $[(6^{-2}) \div (4^{-3})]^{-2}$
24. $(-5)^{-3} \div (125)^{0}$
25. $(\frac{3}{4} + (\frac{1}{2})^{-1})^{-1}$
26. $(5)(6)(5^{-1} + 6^{-1})$
27. $(-3) \div (4^{-1})$
28. $(3^{-1} + 2^{-1})^{-2}$
29. $[3 \cdot (2^{-1}) - 4^{-2}] \div [5(6^{-1}) + (\frac{3}{4})^{-1}]$
30. $[3(5^{-1}) + (\frac{3}{2})^{-1}] \div [(\frac{6}{5})^{-1} - 2(5^{-1})]$

The answers to the previous exercises may be found on page 146.

MULTIPLICATION AND DIVISION WITH EXPONENTS

In this and the next section we will develop three basic rules for working with exponents. These three rules serve as shortcuts in doing numerical computations and are indispensible in simplifying expressions in algebra.

In the following examples we multiply different powers of the same base but we also write our final answers as a single power of the base.

Examples: $2^2 \cdot 2^3 = 4 \cdot 8 = 32 = 2^5$

$2^4 \cdot 2^{-1} = 16 \cdot \frac{1}{2} = 8 = 2^3$

$3^2 \cdot 3^2 = 9 \cdot 9 = 81 = 3^4$

$5^3 \cdot 5^{-3} = (125) \cdot (\frac{1}{125}) = 1 = 5^0$

$6^3 \cdot 6^{-1} = (216) \cdot (\frac{1}{6}) = 36 = 6^2$

Now go back and compare the exponents on the far right with the two exponents at the beginning of the problem on the left. Are you able to make a connection between the three exponents? On careful inspection you should find the exponent in the answer is equal to the sum of the two exponents at the beginning of the problem. This fact is easily justified when the exponents are both positive whole numbers, for in this case each exponent simply counts the number of times the base occurs in the product.

Example: $(3^3)(3^4) = (3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3) = 3^7 = 3^{3+4}$

The same relationship holds when the exponents are zero or negative.

Examples: $5^0 \cdot 5^3 = 1 \cdot 5 \cdot 5 \cdot 5 = 5^3 = 5^{0+3}$

$2^{-2} \cdot 2^{-3} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32} = 2^{-5} = 2^{-2+-3}$

We now state the rule for exponents which says that when multiplying two powers of the same base you can simply write the base once and add the exponents. In the symbolism of algebra we let x denote the base and let m and n denote integer exponents. Our rule then takes the form:

$$x^m \cdot x^n = x^{m+n}$$

It is important to note that the same base must be used if the exponents are to be added. This rule could not be applied to the problem $2^3 \cdot 5^4$ because the base 2 differs from the base 5. The next examples give further illustrations of the rule.

Examples: $3^8 \cdot 3^{-7} = 3^{8+-7} = 3^1 = 3$

$(18)^4 \cdot (18)^{-4} = (18)^{4+-4} = (18)^0 = 1$

$(72)^3 \cdot (72)^{-2} = (72)^{3+-2} = (72)^1 = 72$

$(\frac{3}{4})^{-5} \cdot (\frac{3}{4})^4 = (\frac{3}{4})^{-5+4} = (\frac{3}{4})^{-1} = \frac{4}{3}$

$(26)^{15} \cdot (26)^{-13} = (26)^{15+-13} = (26)^2 = 676$

In each of the above problems it is much simpler to apply the rule for adding exponents rather than multiply out each power individually.

The rule of adding exponents when you multiply has a counterpart when the operation is division. In the following examples, compare the two exponents in each division and try to discover the relationship with the final exponent in the answer.

Examples: $\frac{2^5}{2^2} = \frac{32}{4} = 8 = 2^3$

$3^4 \div 3^2 = 81 \div 9 = 9 = 3^2$

$\frac{4^4}{4^1} = \frac{256}{4} = 64 = 4^3$

The rule for division is to subtract exponents. The order of subtraction is important. We should subtract the bottom or second exponent from the top or first exponent. In algebraic symbolism the rule takes the form:

$$\frac{x^n}{x^m} = x^{n-m} \quad \text{or} \quad x^n \div x^m = x^{n-m}$$

As in multiplication, the same base must be used if the exponents are subtracted. The above rule for division is valid when any exponent is zero or negative. Take special care when subtracting a negative exponent. The following examples are further illustrations of subtracting exponents when the operation is division. Note that both zero and negative exponents appear in these examples.

Examples: $\frac{2^5}{2^{-3}} = 2^{5--3} = 2^{5+^+3} = 2^8$

$4^6 \div 4^{-1} = 4^{6--1} = 4^{6+^+1} = 4^7$

$$\frac{3^0}{3^2} = 3^{0-2} = 3^{-2}$$

$$(\tfrac{2}{3})^4 \div (\tfrac{2}{3})^0 = (\tfrac{2}{3})^{4-0} = (\tfrac{2}{3})^4$$

$$5^5 \div 5^{-5} = 5^{5-^-5} = 5^{5+^+5} = 5^{10}$$

EXERCISES: Use the multiplication and division rules of adding or subtracting exponents to work problems 1-15. Do the same for problems 16-30 except write your final answers to those problems without negative exponents.

1. $3^6 \cdot 3^2$
2. $5^4 \div 5^2$
3. $6^3 \cdot 6^2$
4. $6^2 \div 6^3$
5. $(-2)^4 \div (-2)^7$
6. $0^2 \div 0^1$
7. $(-3)^{-4} \cdot (-3)^{-5}$
8. $(-2)^5 \cdot (-2)^6$
9. $(\frac{5}{6})^6 \cdot (\frac{5}{6})^5$
10. $(\frac{3}{4})^{-2} \div (\frac{3}{4})^{-5}$
11. $(3\frac{1}{2})^2 \cdot (3\frac{1}{2})^{-4}$
12. $(\frac{2}{9})^{-3} \cdot (\frac{2}{9})^3$
13. $(-\frac{4}{5})^0 (-\frac{4}{5})^0$
14. $(-\frac{2}{3})^6 \div (-\frac{2}{3})^6$
15. $2^{-1} \cdot 2^{-2}$
16. $4^3 \div 4^5$
17. $(-7)^0 (-7)^{-2}$
18. $(\frac{3}{2})^2 \div (\frac{3}{2})^3$
19. $(-\frac{1}{2})^3 \div (-\frac{1}{2})^4$
20. $(2\frac{1}{2})^9 \ (2\frac{1}{2})^{-11}$
21. $(-\frac{1}{3}) \div (-\frac{1}{3})^{-3}$
22. $(\frac{1}{4})^0 \div (\frac{1}{4})$
23. $\dfrac{6^2}{6^4}$
24. $\dfrac{5^{-5}}{5^{-3}}$
25. $-\dfrac{8^5}{8^6}$
26. $-\dfrac{3^{-2}}{3^0}$
27. $\dfrac{(-4)^5}{(-4)^7}$
28. $\dfrac{(-3)^6}{(-3)^9}$

29. $\frac{(-3)^4}{(-3)^{-2}}$

30. $\frac{(-2)^5}{(-2)^{-2}}$

The answers to the previous exercises may be found on page 146.

POWERS OF POWERS

The previous section discussed two of the three standard rules for working with exponents. The third rule tells how to operate when one power is followed by another power. Consider the following:

$$(2^2)^3 = (2^2)(2^2)(2^2) = (2^2 \cdot 2^2)(2^2) = (2^4)(2^2) = 2^6$$

The above problem is an example of a double power, or what may be called a power of a power. The power inside the parentheses is a second power or a square. The power outside the parentheses is a third power or a cube and applies to the whole expression in parentheses. As a first step, we wrote the expression in parentheses 3 times. Next, we rearranged the powers of 2 and then applied the addition rule for exponents to get 2^4 times 2^2. Finally, we applied the addition rule for exponents a second time to get a final answer of 2^6. Can you think of a relationship between the beginning two exponents, 2 and 3, and the final exponent in the answer which is 6?

Now consider the following example:

$$(4^3)^5 = (4^3)(4^3)(4^3)(4^3)(4^3) = 4^{3+3+3+3+3} = 4^{15}$$

In this example, we first apply the outer exponent 5 which tells us to write the expression in parentheses 5 times. Then, we apply the addition rule for exponents several times. This amounts to adding all the exponents. Of course, each exponent is a 3 and there are five of them to be added. If we again compare the beginning two exponents, 3 and 5, with the final exponent in the answer, 15, we can begin to guess a rule for powers of powers.

Multiplication is a shortcut for repeated addition and the second example above shows why repeated addition occurs when the problem involves a double power. We formally write the rule for simplifying a power of a power as:

$$(x^m)^n = x^{m \cdot n}$$

This is our third and final rule for working with exponents. Notice how this rule differs from the rule for multiplying two different powers of x.

$$(x^m)(x^n) = x^{m+n}$$

Here the base x is written twice. But when a power of a power is involved, the base x is written only once and the two exponents are multiplied instead of added.

The rule of multiplying exponents when evaluating a power of a power is valid when either or both exponents are zero or negative. The following examples are further illustrations of this third rule for

exponents.

Examples: $(2^{-3})^4 = (2^{-3})(2^{-3})(2^{-3})(2^{-3}) = 2^{-3+-3+-3+-3} = 2^{-12}$

$$(5^3)^{-2} = \frac{1}{(5^3)^2} = \frac{1}{(5^3)} \cdot \frac{1}{(5^3)} = \frac{1}{5^{3+3}} = \frac{1}{5^6} = 5^{-6}$$

$$(4^{-2})^{-3} = \frac{1}{(4^{-2})^3} = \frac{1}{(4^{-2})} \cdot \frac{1}{(4^{-2})} \cdot \frac{1}{(4^{-2})} = \frac{1}{4^{-2+-2+-2}} = \frac{1}{4^{-6}} = 4^6$$

$$(2^0)^3 = (2^0)(2^0)(2^0) = 2^{0+0+0} = 2^0 = 1$$

In each of the above examples you should note how the final exponent answer can be obtained by simply multiplying the two beginning exponents.

For the sake of being complete, we repeat below the 3 standard rules for operating with exponents.

$$x^m \cdot x^n = x^{m+n}$$

$$x^m \div x^n = x^{m-n} \quad \text{or} \quad \frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{m \cdot n}$$

EXERCISES: Use the three standard rules for exponents to simplify problems 1-30. Your final answers should be written without using negative numbers as exponents.

1. $(3^2)^5$
2. $[(-2)^3]^2$
3. $(4^{-1})^5$
4. $(7^2)^{-4}$
5. $(3^{-2})^{-4}$
6. $[(-3)^{-4}]^{-5}$
7. $[(-2)^3]^{-2}$
8. $(3^4)^{-5}$
9. $(2^{-2}) \cdot (2^3)$
10. $(3^{-5}) \div (3^4)$
11. $[(3^2)(3^{-5})]^2$
12. $[(2^3)(2^{-7})]^{-4}$

13. $[(\frac{2}{3})^{-3}]^2$

14. $[(3\frac{1}{2})^2]^3$

15. $[(\frac{4}{3})^{-1}]^{-2}$

16. $[(\frac{1}{2})\cdot(\frac{1}{2})^3]^{-5}$

17. $[(\frac{3}{8})^0(\frac{3}{8})^3]^0$

18. $[(-2\frac{2}{3})^0]^{-6}$

19. $[4^3 \div 4^5]^{-2}$

20. $\left[\frac{6^{-5}}{6^{-3}}\right]^{-2}$

21. $\left[\frac{3^0}{3^{-2}}\right]^{-3}$

22. $\left[\frac{(-2)^5}{(-2)^{-3}}\right]^{-4}$

23. $[(2^{-3})(2^3)]^6$

24. $[(3^{-4}) \div (3^{-5})]^2$

25. $\left[(-2)^{-2}(-2)^{-2}\right]^{-2}$

26. $\left[(0^2)(2^0)\right]^3$

27. $[(-2)^3]^{-2} \div [(-2)^2]^3$

28. $(-3)^{-4}(-3)^{-5} \div (-3)^2$

29. $[(2^3) \div (2^5)(2^{-6})]^{-2}$

30. $\frac{(6^2)^3 \cdot (6^{-5})^2}{(6^{-1})^{-2}(6^{-3})^{-4}}$

The answers to the previous exercises may be found on page 147.

DECIMALS AND POWERS OF 10

In this section we will examine the relationships between decimals and powers of 10. Numbers which are written in decimal form are said to be in ordinary notation. The following list shows various nonnegative powers of 10 converted to ordinary notation.

$10^2 = 100$ $\quad$ $10^1 = 10$

$10^3 = 1{,}000$ $\quad$ $10^0 = 1$

$10^4 = 10{,}000$ $\quad$ $10^6 = 1{,}000{,}000$

Reviewing the above list reveals a direct relationship between the exponents and the number of zeros written after the digit 1 when the powers of 10 are converted to ordinary notation. The number of zeros is equal to the exponent.

Now consider the following list of negative powers of 10 converted to ordinary notation.

$10^{-1} = 0.1$ $\quad$ $10^{-4} = 0.0001$

$10^{-2} = 0.01$ $\quad$ $10^{-5} = 0.00001$

$10^{-3} = 0.001$ $\quad$ $10^{-6} = 0.000001$

This list indicates that when negative exponents are involved the number of zeros between the decimal point and the digit 1 is one less than the absolute value of the exponent.

Another way to consider the relationship between powers of 10 and ordinary notation is to think of each power of 10 controlling the shifting of the decimal point. In this case it is convenient to start with the number "1." and shift the decimal point to the right or to the left of the digit 1, depending on whether the exponent is positive or negative. If the exponent is positive we shift right and if the exponent is negative we shift left. The number of decimal places that are shifted is always equal to the absolute value of the exponent.

Examples: 10^5 Start with 1. and shift the decimal point 5 places to the right.

$10^5 = 100000. = 100000.$

10^{-4} Start with 1. and shift the decimal point 4 places to the left.

$10^{-4} = .0001 = 0.0001$

The idea of thinking of the power of 10 as controlling the shifting of the decimal point proves useful when any decimal number is multiplied by a power of 10. All we need remember is that a positive exponent indicates the shift will be to the right, a negative exponent indicates the shift should be to the left. Of course, if the exponent is zero then no shifting takes place since $10^0 = 1$ and multiplying any number by 1 does not change the number.

Examples:	$2.361 \times 10^2 = 236.1$	Shift decimal point	2 places to the right.
	$476.25 \times 10^{-3} = 0.47625$	Shift decimal point	3 places to the left.
	$-0.0096845 \times 10^4 = -96.845$	Shift decimal point	4 places to the right.
	$0.0261 \times 10^{-2} = 0.000261$	Shift decimal point	2 places to the left.

EXERCISES: In problems 1-20 convert each number to ordinary notation. Convert the numbers in problems 21-30 to single powers of 10. Perform the additions and subtractions in problems 31-40 by first converting the numbers to ordinary notation.

1. 10^4
2. 10^{-3}
3. 10^7
4. 10^{-7}
5. 10^{-1}
6. -10^{-2}
7. 10^0
8. -10^5
9. 10^{-6}
10. 10^{-4}
11. 12.36×10^{-1}
12. 158.42×10^2
13. 0.0001368×10^5
14. 0.0219×10^{-2}
15. 4.37×10^0
16. 0.0024×10^{-3}
17. -1869.3×10^4
18. -14.7×10^{-4}
19. 19.2×10^5
20. 0.016×10^{-3}
21. 0.0010
22. 10,000
23. 0.01
24. 100,000
25. 1,000,000,000
26. -0.0001

27. 0.000100

28. 0.1

29. 0.0000001

30. 1000.00

31. $(1.29\text{x}10^{2}) + (346.31\text{x}10^{-1})$

32. $(4.7\text{x}10^{-1}) - (2.36\text{x}10^{1}) + (47.96\text{x}10^{-1})$

33. $(42.69\text{x}10^{2}) - (-12.16\text{x}10^{3})$

34. $(12.3\text{x}10^{2}) + (-146.2\text{x}10^{-1})$

35. $(9\text{x}10^{3}) - (9\text{x}10^{2}) - (9\text{x}10^{1})$

36. $(0.00631\text{x}10^{3}) + (528\text{x}10^{-4})$

37. $(7.2\text{x}10^{3}) - (4.1\text{x}10^{0}) + (1.61\text{x}10^{-2})$

38. $(2.3\text{x}10^{3}) + (3.1\text{x}10^{2}) - (4.5\text{x}10^{-1})$

39. $(0.1\text{x}10^{-1}) + (0.1\text{x}10^{-2}) + (0.1\text{x}10^{-3})$

40. $(8.7\text{x}10^{3}) - (-127.4\text{x}10^{1}) - (1476\text{x}10^{-3})$

The answers to the previous exercises may be found on page 147.

SCIENTIFIC NOTATION

One way of expressing ordinary numbers using powers of 10 has become standard and is used in every scientific field. The system of notation is called scientific notation. This system is particularly useful for describing very large or very small numbers. It also provided a convenient way of performing multiplications and divisions on slide rules. Although slide rules are now obsolete, all scientific calculators and computers have provisions for displaying or printing numbers in scientific notation.

Any number written as a power of 10 times a decimal number whose absolute value is greater than or equal to 1 but less than 10 is said to be written in <u>scientific notation.</u>

Examples: $43 = 4.3 \times 10^{1}$

$-279 = -2.79 \times 10^{2}$

$0.00248 = 2.48 \times 10^{-3}$

The three examples above show the numbers in ordinary notation to the left of the "=" sign and show the same numbers to the right of "=" sign in scientific notation.

Another way of describing scientific notation is to require exactly one nonzero digit to the left of the decimal point in the number which is multiplied by some power of 10. The following are examples of numbers which are <u>not</u> in scientific notation.

Examples:

26.34×10^{2}	should be 2.634×10^{1}
-0.125×10^{-3}	should be -1.25×10^{-1}
$4. \times 5^{2}$	should be 1.00×10^{2}
0.00693×10^{4}	should be 6.93×10^{1}
10.0×10^{4}	should be 1.00×10^{5}

In the previous section we used examples of numbers in scientific notation. In this section our primary concerns are learning to convert numbers to scientific notation and learning how to multiply and divide numbers that are in scientific notation.

To convert a number to scientific notation, first write the number in ordinary notation, then place the decimal point to the right of the first nonzero digit in the number. This should result in a number between 1 and 10. Finally, write this number (with negative sign if necessary) times a power of 10 which will shift the decimal point back to the position it was in when the number was written in ordinary

notation.

Examples: Change the following numbers to scientific notation.

427.89×10^{-1} first write in ordinary notation 42.789
next write, 4.2789
then finally write 4.2789×10^{1}

-0.00123×10^{6} in ordinary notation this is -1230.
next write, -1.230
then finally write -1.230×10^{3}

When numbers written in scientific notation are to be added or subtracted, they should first be converted to ordinary notation and then operated on as in the problems at the end of the exercises in the previous section. But the following shortcut may be applied when numbers in scientific notation are to be multiplied or divided.

To multiply or divide two numbers in scientific notation, multiply or divide their numerical parts and their powers of 10 separately, then convert the final answer to scientific notation.

Examples: Multiply: $(2.36 \times 10^{2})(4.1 \times 10^{-3})$

First, re-arrange the products as:

$$(2.36 \times 4.1)(10^{2} \times 10^{-3})$$

Then, multiply the numerical parts and add the powers of 10.

$$(9.676)(10^{-1}) = 0.9676$$

Finally, convert back to scientific notation for the final answer.

$$9.676 \times 10^{-1}$$

Divide: $\dfrac{1.4518 \times 10^{2}}{4.27 \times 10^{-1}}$

First, re-arrange the problem as:

$$\frac{1.4518}{4.27} \cdot \frac{10^{2}}{10^{-1}}$$

Then, divide the numerical parts and subtract the bottom power of 10.

$$(0.34)(10^{2--1}) = (0.34)(10^{3}) = 340.$$

Finally, convert back to scientific notation for the final answer.

3.4×10^2

EXERCISES: Perform the operations in problems 1-25 and leave your final answers in scientific notation.

1. $2.37 \times 10^2 + 4.1 \times 10^1$

2. $4.679 \times 10^3 - 5.5 \times 10^5$

3. $56.92 \times 10^{-1} + 3.4 \times 10^{-2}$

4. $-1.2 \times 10^{-1} - 4.6 \times 10^{-1}$

5. $4.9 \times 10^3 - 2.2 \times 10^{-1}$

6. $-8.2 \times 10^2 + 1.4 \times 10^1$

7. $(-4.9 \times 10^3) - (-3.21 \times 10^2)$

8. $(-2.1 \times 10^{-1}) - (-4.6 \times 10^{-2})$

9. $(2.4 \times 10^2)(3.9 \times 10^3)$

10. $(5.429 \times 10^2)(2.0 \times 10^{-4})$

11. $(6.84 \times 10^5) \div (4.0 \times 10^{-4})$

12. $(-6.8 \times 10^{-2})(-2.1 \times 10^3)$

13. $(9.2 \times 10^{-1}) \div (4.6 \times 10^{-4})$

14. $(-2.9 \times 10^2)(3.07 \times 10^{-8})$

15. $(-8.48 \times 10^{-3}) \div (-8.0 \times 10^{-5})$

16. $(-7.23 \times 10^{-4})(-5.6 \times 10^{-4})$

17. $\dfrac{6.84 \times 10^{5}}{6.0 \times 10^{-4}}$

18. $\dfrac{(4.9 \times 10^{-2})(2.2 \times 10^{5})}{(7.7 \times 10^{-6})}$

19. $\dfrac{(3.6 \times 10^{2})(-6.8 \times 10^{3})}{(5.1 \times 10^{-6})}$

20. $\dfrac{(4.2 \times 10^{-2})(-2.5 \times 10^{-7})}{(-3.5 \times 10^{-5})}$

21. $\dfrac{(3.6 \times 10^{2}) + (1.2 \times 10^{2})}{(2.4 \times 10^{2})}$

22. $\dfrac{(7.2 \times 10^{-3})(5.1 \times 10^{-6})}{(4.0 \times 10^{5})(3.0 \times 10^{-8})}$

23. $\dfrac{(5.6 \times 10^{2}) \div (2.8 \times 10^{3})}{(3.0 \times 10^{1}) \div (2.4 \times 10^{0})}$

24. $\dfrac{(2.4 \times 10^{2})(3.0 \times 10^{-3})}{(2.9 \times 10^{1}) + (2.1 \times 10^{1})}$

25. $\dfrac{(1.8 \times 10^{-5})(3.0 \times 10^{-6})}{(3.6 \times 10^{3}) \div (1.2 \times 10^{-4})}$

The answers to the previous exercises may be found on page 148.

SECTION V REVIEW PROBLEMS

Simplify problems 1-10 by writing the answers without negative exponents.

1. 3^{-4} 2. 5^{-2} 3. $(-2)^{-3}$ 4. $(\frac{3}{4})^{-2}$ 5. $(2\frac{1}{2})^{-3}$

6. $(-3)^{-2}$ 7. 0^{-1} 8. $(\frac{5}{7})^{-2}$ 9. $(-\frac{3}{5})^{-3}$ 10. $(-1)^{-37}$

Perform the indicated operations in problems 11-20.

11. $(-2)^{-3} + (-3)^{-2}$

12. $(2 + -3)^{-2}$

13. $6 \cdot (3)^{-2} + (\frac{3}{2})^{-1}$

14. $(\frac{2}{3})^{2} - (\frac{3}{2})^{-3}$

15. $(6)^{-3} \div (17)^{0}$

16. $(4) \cdot (3) \cdot [4^{-1} + 3^{-1}]$

17. $[(\frac{1}{2})^{-1}] \div [(\frac{3}{5})^{-2}]$

18. $(\frac{5}{7} + (-\frac{1}{2})^{-3})^{-1}$

19. $[8 \cdot (2^{-4}) - 2^{-1}] \div [3^{-2} + (-2)^{3}]$

20. $[(\frac{3}{4})^{-1} \div (\frac{1}{2})^{-2}]^{-1} \div [(\frac{1}{3})^{-1} \div (\frac{2}{5})^{-2}]^{-1}$

Use the multiplication and division rules for exponents to work problems 21-35.

21. $4^{2} \cdot 4^{3}$ 22. $5^{2} \div 5^{3}$ 23. $(-2)^{6} \div (-2)^{8}$

24. $(-3)^{6} \cdot (-3)^{-4}$ 25. $(\frac{2}{3})^{4} \div (\frac{2}{3})^{-5}$ 26. $4^{17} \div 4^{12}$

27. $(-5)^{20} \div (-5)^{20}$ 28. $(-2)^{7} \div (-2)^{0}$ 29. $2^{5} \cdot 2^{6} \cdot 2^{3}$

30. $\dfrac{6^{5}}{6^{7}}$ 31. $\dfrac{2^{4}}{2^{-5}}$ 32. $\dfrac{(-3)^{6}}{(-3)^{-9}}$

33. $(-2)^{4} \cdot (-2)^{6} \cdot (-2)^{3}$

34. $5^{2} \div 5^{5} \div 5^{8}$

35. $3^{6} \cdot 3^{-4} \div 3^{-2}$

Use the three standard rules for exponents to simplify problems 36-45. Your final answers should be written without using negative numbers as exponents.

36. $(2^3)^4$

37. $(5^2)^{-3}$

38. $(3^4)^{-5}$

39. $(4^3)(4^2)$

40. $(6^{-2})^7$

41. $\left[\left(\frac{3}{5}\right)^{-2}\right]^4$

42. $[2^3 \cdot 2^4]^{-3}$

43. $[7^3 \div 7^5]^{-3}$

44. $[3^2 \div 3^4 \cdot 3^5]^{-2}$

45. $\dfrac{(2^3)^2 \div (2^4)^3}{(2^{-1})^3 \cdot (2^3)^{-1}}$

Convert the powers of 10 in problems 46-55 to ordinary notation.

46. 10^3 47. 10^{-2} 48. 10^7 49. 10^4 50. 10^{-3}

51. 10^{-4} 52. 10^5 53. 10^{-5} 54. 10^{-1} 55. 10^{-6}

Convert the decimals in problems 56-64 to powers of 10.

56. 10,000 57. 1,000,000 58. 0.01

59. 0.0001 60. 0.1 61. 1,000

62. 0.00001 63. 0.0000001 64. 0.001

Perform the operations in problems 65-75 and leave your answers in scientific notation.

65. $3.65 \times 10^2 + 4.1 \times 10^3$

66. $-3.98 \times 10^1 - 5.6 \times 10^{-2}$

67. $8.1 \times 10^3 - 5.4 \times 10^5$

68. $(2.9 \times 10^3)(4.5 \times 10^2)$

69. $(-3.71 \times 10^4)(4.0 \times 10^{-6})$

70. $(12.818 \times 10^2) \div (2.6 \times 10^{-5})$

71. $(-5.7 \times 10^{-3})(-2.6 \times 10^{-5})$

72. $(-6.5116 \times 10^{-2}) \div (-8.92 \times 10^{-8})$

73. $\dfrac{(4.2 \times 10^5)\ (1.2 \times 10^6)}{(2.1 \times 10^{-3})}$

74. $\dfrac{(8.4 \times 10^2) - (6.3 \times 10^1)}{(1.11 \times 10^{-2})}$

75. $\dfrac{(7.4 \times 10^{2})\ (3.0 \times 10^{-3})}{(1.5 \times 10^{1})\ (3.7 \times 10^{-6})}$

The answers to the Review Problems may be found on pages 149 and 150.

SECTION VI RUDIMENTS OF ALGEBRA

Like Terms

In this section we will develop some of the basic rules for operating on the terms or expressions used in algebra. We also introduce a minimum of vocabulary from algebra.

Algebra is only one branch of the subject of mathematics which is a systematic and logical approach to the study of complex problems. As a language, algebra can be used to express complex relationships between numbers and also to symbolically express relationships between physical quantities. One of the most important ideas to be learned in algebra is the use of equations to solve problems.

Problem: A movie theatre charges \$2.25 for children and \$4.75 for adults. As the manager of the theater your problem is to determine how many children and how many adults attended last nights showing. The only information you are given is that 480 seats were filled and the total amount of money collected was \$1600.00.

You may consider the above problem as a puzzle for now. You might even try to guess the answer. But after several attempts at guessing you may become curious as to how to approach the problem systematically. The language of algebra is what is needed to solve the above problem and many others just like it.

Our immediate concern is not to learn how to solve problems like the one above. Such problems are really part of formal algebra. But to be successful with algebra requires a good foundation in the rudiments of algebra and this section is concerned with providing that foundation. You may consider the previous sections which dealt with positive and negative numbers, the order of operations, fractions and decimals, and rules for exponents, as providing pre-natal conditioning. We will now begin to do formal algebra. You will learn how to set up and solve equations in the course called elementary algebra.

A variable is a symbol which may be used to stand for any one of a particular collection of numbers. In algebra the symbols we use as variables are simply single letters of the alphabet such as x and y. However, there are some symbols used to represent numbers which are not variables. For example, consider the formula which gives the area A of a circle in terms of its radius r.

$$A = \pi r^2$$

The variables in this formula are A and r. But the symbol "π" is called "pi" and is not a variable. π is not a variable because it doesn't vary, it always represents the same number whose value rounded to 10 digits is:

$$\pi \cong 3.141592654$$

π is called a constant. In general, a <u>constant</u> is a variable which can take only one value.

A <u>term</u> is a mathematical expression using numerals or variables or both to indicate a product or a quotient. For example, the right side of the above equation consists of the term πr^2 in which π is the constant and r is the variable. Examples of terms include:

$$7xy \quad \frac{x}{a-b} \quad 8x^3y^2 \quad \frac{4a + 8b}{3c + 2d}$$

A <u>monomial</u> is an expression which is a product of a number and powers of one or more variables. As examples, the following are all monomials.

$$3x \quad 2y^2 \quad 5x^2y^3z^4 \quad a^3b^2$$

A <u>polynomial</u> is an indicated sum of monomials. A sum of two monomials is called a <u>binomial</u>, and a sum of three monomials is called a <u>trinomial</u>.

Examples: $3x^2 + 5y^3$ (binomial)

$-4x^3 + 8y^4 - 14z^5$ (trinomial)

$15x^4 - 7x^3y^2 + 2xy + 7y^4$ (polynomial)

So the words "binomial" and "trinomial" simply refer to special cases of polynomials, namely polynomials consisting of two (binomial) or three (trinomial) terms.

It is essential to understand the meaning and the order of the operations of algebraic expressions written using variables. For example, when we write:

$$3x^2$$

the exponent on the variable x stands for the squaring operation. Just as important is that there is an implied multiplication operation between the 3 and x^2. So the meaning of $3x^2$ is to first square x and then multiply that result by 3.

Now consider the expression $(3x)^2$. The exponent 2 is outside the parentheses so the intended meaning here is to first multiply x by 3 and then square that product. So although the two expressions:

$$3x^2 \quad \text{and} \quad (3x)^2$$

may sound the same when we read them, there is a real difference in meaning.

When we write the term $5a$ the sign on the 5 is understood to be $+$ and the exponent on a is understood to be 1. Thus,

$$5a = +5a^1$$

but we simplify the amount of writing if we use the expression on the left. Similarly, when we write monomials in which the beginning number is a 1 we simplify matters by dropping the 1 from the term. Thus, the following terms have the indicated meanings.

$$x = 1 \cdot x \qquad a^2b^3 = 1 \cdot a^2b^3 \qquad -xyz^2 = -1 \cdot xyz^2$$

The numerical part of a term is called the coefficient of the remaining part of the term. The following examples illustrate the coefficient concept.

Examples:

$3x^2y^4$	the coefficient of x^2y^4 is 3
$8a$	the coefficient of a is 8
$-z^3$	the coefficient of z^3 is -1
$-5x^3y^5$	the coefficient of x^3y^5 is -5
xy^4	the coefficient of xy^4 is 1

Finally, we define what are called like terms. Two terms are called like terms if they are exactly alike or if they differ only in their numerical coefficients. So like terms must have the same variables with the same powers on those variables.

Examples:

$5x^2y^3$	and	$-2x^2y^3$ are like terms
$5x^2y^3$	and	$5a^2b^3$ are not like terms
$-3p^2q^2r^2$	and	$40p^2q^2r^2$ are like terms
$-3p^2q^2r^2$	and	$-3p^2q^3r^2$ are not like terms
$17x^2y^5z^4$	and	$17x^2y^5$ are not like terms

Sums and differences of like terms can be replaced by a single term by simply adding or subtracting the numerical coefficients. This rule applies only to like terms.

Examples:

$$2x + 3x = 5x$$

$$8y^2 - 4y^2 = 4y^2$$

$$17a^2b^3 - 23a^2b^3 = -6a^2b^3$$

$$x^2y^4 + 3x^2y^4 - 5x^2y^4 = -x^2y^4$$

Note that when adding or subtracting like terms only the numerical coefficients are added or subtracted. The exponents on the variables do not change.

A natural question to ask is how one would add unlike terms. The answer for now at least is you cannot add or subtract unlike terms. When we write $2x + 3y$ we are only indicating that we should add $2x$ and $3y$. The expression makes sense because x and y are variables that represent numbers. As soon as we know the numerical values for x and y we can multiply x by 2, then multiply y by 3 and then add the two products. Thus, if $x = 5$ and $y = 7$ then $2x + 3y = 2 \cdot 5 + 3 \cdot 7 = 10 + 21 = 31$.

To say that you cannot add or subtract unlike terms only means that two such terms cannot be combined to make a single term that is similar to the original two terms.

There are times when we would like to add or subtract like terms in expressions which contain parentheses as grouping symbols. As was shown earlier, the distributive law can be used to insert or delete grouping symbols. In the next part we will discuss the distributive law in more detail. For now, we present two rules for simplifying expressions which contain grouping symbols.

1. If the operation in front of the left parentheses is $+$, then simply drop both left and right parentheses.
2. If the operation in front of the left parentheses is $-$, then change the operation to $+$ and change the sign on the numerical coefficient of each term within the parentheses before dropping both left and right parentheses.

Examples: $2x^2 + (3x + 5x^2) + 7x$

$= 2x^2 + 3x + 5x^2 + 7x = 7x^2 + 10x$

$5ab + 2c - (8ab - 6c)$

$= 5ab + 2c + {}^-8ab - {}^-6c$

$= {}^-3ab + 2c + {}^+6c = -3ab + 8c$

$6p + 8q - 5r - (9p + 14q - 3r)$

$= 6p + 8q - 5r + {}^-9p + {}^-14q - {}^-3r$

$= -3p + {}^-6q - 5r + {}^+3r$

$= -3p - 6q - 2r$

EXERCISES: Simplify the following expressions by combining like terms.

1. $15x + 6x$
2. $9a + {}^{-}6a$
3. $32y - 18y$
4. ${}^{-}24z + 52z$
5. $24x^2 - 42x^2$
6. $a^2b + 5a^2b$
7. $23m^2 - m^2$
8. $t^3 - t^3$
9. $w^4 + 5w^4$
10. $12x + 3x + 18x$
11. $14ab^2 - 21ab^2 + 16ab^2$
12. $19x^2 + {}^{-}31x^2 + 15x^2 + {}^{-}12x^2$
13. $a^3b + {}^{-}5a^3b + {}^{-}9a^3b - {}^{-}5a^3b$
14. $3x + 2y + 9x + 8y$
15. $5m^2 + 6 + {}^{-}10m^2 - 4$
16. $34d^2 - 19 + {}^{-}6d^2 - 12$
17. $34x^2 - 16y^2 - 52x^2 + 30y^2$
18. $9g + 8h + 12h - 13g$
19. $2x^2 + 3x - 9x^2 + 8x$
20. $5y - 12y^2 + 3y + y^2$
21. $13xy + 12xy - 42xy$
22. $8xy - x + 3y - 17xy + 4y + 12xy - 31x$
23. $5a^2b + 3ab^2 - 8a^2b + 12ab^2 + 7a^2b - 13ab^2$
24. $-9x^2 + 4x^3 - 17x + 2x^2 - 8x + 9x^3 - 4x + 13x^2$
25. $3x^2 \cdot x^3 + 4x \cdot x^4 - 21x^3 \cdot x^2 + 18x^5 - 14x^4 \cdot x$
26. $17y^3 - (4y^3 - 11y^2 + 4y) + 3y^2 - 9y$
27. $42zw^2 + 13zw - (21zw^2 - 8zw - z^2w) + 17z^2w$
28. $-18xy^3 + 12xy + (15xy^3 - 13xy) - ({}^{-}6xy - 3xy^3)$
29. $w^6 - 5ww^5 - (w^3 \cdot w^3 - 12w^2w^4 - 29w^4w^2)$
30. $17x^3y^2 - ({}^{-}x^2y^3 - 23x^3y^2 - 19x^2y^3) + 18x^2y^3$
31. $8x^2 - 3x - 7 - (16x^2 - 8x + 4) + 3x^2 + 2x - 5$
32. $5a^6 - 8a^5 + 2a^4 - 3a^2 + 21a - (8a^6 - 13a^5 - 5a^4 + 2a^3 - 7a)$
33. $-14x^2y - (3x^2y - 7 + 26x^2y) + ({}^{-}x^2y - 12 + 3x^2y)$
34. $({}^{-}27x^3y^4 - 12x^4y^3) - (31x^4y^3 - 13x^3y^4) - ({}^{-}2x^3y^4 - 19x^4y^3)$

35. $7w^3 - 4w^2 + 5w - 3 - (6 - w + 3w^2 - 8w^3)$

The answers to the previous exercises may be found on page 151.

Multiplication by Monomials

In the previous section we learned how to add and subtract monomials which were like terms. We emphasized that addition and subtraction make sense only when we have the same variables written to the same powers. In this part we will learn how to multiply any two monomials and how to multiply a monomial times a polynomial.

Although we can't combine $5x^2y^3 + 3xy^2$ to make a single term, we can multiply these two terms and write their product as a single term. We do this by re-arranging the parts of the product as indicated below. Earlier we mentioned that the associative and commutative laws are valid for multiplication, but not for subtraction or division. The associative and commutative laws for multiplication justify re-arranging the parts of the product.

$$(5x^2y^3)\cdot(3xy^2) = (5\cdot 3)(x^2\cdot x)(y^3\cdot y^2)$$

$$= (15)(x^3)(y^5)$$

$$= 15x^3y^5$$

Note that in re-arranging the parts to be multiplied we put the numbers together and put the same variables together. Then when we actually multiplied, we multiplied the numbers and variables separately. When multiplying variables we use the addition rule for exponents. We don't require parentheses in the final answer.

When multiplying any two monomials we should keep in mind the following order:

SIGNS NUMBERS VARIABLES IN ALPHABETIC ORDER

Study the next examples and note how the final answers correspond to the above order.

Examples:

$$(4x^3y^6)(7x^2y) = (4\cdot 7)(x^3\cdot x^2)(y^6\cdot y) = 28x^5y^7$$

$$(-3a^2b^3)(4a^2b^4) = (-3\cdot 4)(a^2\cdot a^2)(b^3\cdot b^4) = -12a^4b^7$$

$$(-6x^2z^3y^4)(2y^3x^2z) = (-6\cdot 2)(x^2\cdot x^2)(y^4\cdot y^3)(z^3\cdot z) = -12x^4y^7z^4$$

$$(-5b^3a)(-7a^4)(2b^5) = (-5)(-7)(2)(a\cdot a^4)(b^3\cdot b^5) = 70a^5b^8$$

$$(9a^2y^3x^2)(-4y^5x^2a^3) = (9)(-4)(a^2\cdot a^3)(x^2\cdot x^2)(y^3\cdot y^5) = -36a^5x^4y^8$$

The above examples show how to multiply two monomials and we would now like to extend multiplication to include multiplying a monomial times a polynomial. The polynomial will always be written in parentheses and the monomial we multiply with will usually be written

on the left, although it may sometimes be written on the right.

The distributive law tells us that when we have a sum of monomials in parentheses and are multiplying by another monomial outside the parentheses we can simply multiply the outer monomial times each monomial within the parentheses. Then we add all the products of the monomials. The next example illustrates the multiplication process using the distributive law.

Example: $5x^2 \cdot (3x^4 + 2x^3 + 7x^2)$

$= 5x^2 \cdot 3x^4 + 5x^2 \cdot 2x^3 + 5x^2 \cdot 7x^2$

$= 15x^6 + 10x^5 + 35x^4$

In applying the distributive law in the above example the net effect was to distribute the outer term times each inner term. Remember that in algebra when we write two terms next to each other there is an implied operation of multiplication between the terms. Sometimes the distributive law is written in a symbolic form such as:

In the above example $5x^2$ would take the place of the $\triangle$, $3x^4$ would take the place of the $\bigcirc$, $2x^3$ would take the place of the $\bigcirc$ and $7x^2$ would replace the $\square$. Here we have a sum of three terms within the parentheses but the distributive law can be applied to any number of terms within parentheses.

The next examples give further illustrations of the distributive law.

Examples: $a^3(2a^4 + 6a^5 + 4a^2)$

$= a^3 \cdot 2a^4 + a^3 \cdot 6a^5 + a^3 \cdot 4a^2$

$= 2a^7 + 6a^8 + 4a^5$

$2xy^2(3x^3y + 9x^2y^3 + 5xy^2)$

$= 2xy^2 \cdot 3x^3y + 2xy^2 \cdot 9x^2y^3 + 2xy^2 \cdot 5xy^2$

$= 6x^4y^3 + 18x^3y^5 + 10x^2y^4$

Finally, we note that the distributive law also works when the terms within the parentheses are subtracted instead of added. The monomial outside the parentheses may also contain a negative number as a coefficient. In this case it is best to enclose the outer term within individual parentheses so as not to confuse the negative sign

with the subtraction operation. The next examples illustrate how to operate when subtraction and negative signs are involved. We also include an example of multiplying on the right.

Examples: $(2z^3)(6xz - 4y^2z^2 - 8xyz^3)$

$= 2z^3 \cdot 6xz - 2z^3 \cdot 4y^2z^2 - 2z^3 \cdot 8xyz^3$

$= 12xz^4 - 8y^2z^5 - 16xyz^6$

$(-3ab^2)(2a^3b + 6a^2b + 11ab)$

$= (-3ab^2) \cdot 2a^3b + (-3ab^2) \cdot 6a^2b + (-3ab^2) \cdot 11ab$

$= -6a^4b^3 + -18a^3b^3 + -33a^2b^3$

$= -6a^4b^3 - 18a^3b^3 - 33a^2b^3$

$(9x^2y^3 - 2x^3y^5 + 6xy^2) \cdot (3x^2y^2)$

$= 9x^2y^3 \cdot 3x^2y^2 - 2x^3y^5 \cdot 3x^2y^2 + 6xy^2 \cdot 3x^2y^2$

$= 27x^4y^5 - 6x^5y^7 + 18x^3y^4$

$(-5a^2bc^3)(2ab^2c - 4ab + 5ac - 3)$

$= (-5a^2bc^3) \cdot 2ab^2c - (-5a^2bc^3) \cdot 4ab + (-5a^2bc^3) \cdot 5ac - (-5a^2bc^3) \cdot 3$

$= -10a^3b^3c^4 - (-20a^3b^2c^3) + (-25a^3bc^4) - (-15a^2bc^3)$

$= -10a^3b^3c^4 + 20a^3b^2c^3 - 25a^3bc^4 + 15a^2bc^3$

EXERCISES: Multiply the following terms. Use the distributive law where needed and add or subtract like terms whenever possible.

1. $(3x^2)(6x^3)$
2. $(-4a^3)(2a)$
3. $(-5x^4)(-4x^5)$
4. $(3ab^2)(-7a^3b)$
5. $(-9xy^3)(-2x^3y^3)$
6. $(-14xy^2z^3)(3x^3yz^2)$
7. $6x^5y^2 \cdot 4xy^3$
8. $(-8a^2b^3) \cdot 4ab$
9. $6w \cdot 3w^5 + 2w^2 \cdot 4w^4$

10. $-5x^2 \cdot 4x^5 + 8x^3 \cdot 3x^4$

11. $-6a \cdot 9a^6 + 4a^4 \cdot 3a^3 - 9a^5 \cdot a^2$

12. $19x^3 \cdot 2x - 6x^2 \cdot 4x^2 - 7x \cdot 2x^3$

13. $-5w^5 \cdot 6w^4 - 3w^2 \cdot 8w^7 + 4w^6 \cdot 3w^3$

14. $(-6z^2)(-3z^4) - (5z^3)(-2z^3) + (4z^5)(-3z)$

15. $6x^2y \cdot 4x^3y^5 - 3x \cdot 7x^4y^5 + 9xy^3 \cdot 4x^4y^2$

16. $3a^2b^3 \cdot 2a^3b + 8ab \cdot 2a^2b^5 - 4a^3b^2 \cdot 3a^2b^2 + 2a^2b^3 \cdot 5ab^3$

17. $7y^3z^3 \cdot 3z^3y^2 - 4y^4z^2 \cdot 5y^1z^3 + 2y^3z^2 \cdot 6y^2z^4 - 12y^2z^3 \cdot 2z^2y^3$

18. $9x^3y^5 \cdot 2x^4y^1 - (7x^4y^2 \cdot 3x - 4x^2y \cdot 5x^5y^5) + 2xy \cdot 6x^4y$

19. $6a^4b \cdot 3a^3b^2 - (8a^5b^4 \cdot 3a^2b - 9a^2b^2 \cdot 2a^5b) - 5ab^3 \cdot 3a^6b^2$

20. $4w^3(2w + 5w^2 + 7w^3)$

21. $9a^2(3a^3 - 2a + 4a^2)$

22. $(-3x)(5x + 6x^2 - 8x^3)$

23. $x^3y^2(xy^4 + x^2y^3)$

24. $4a^3b^4(2ab - 9a^2b + 3a^3b^2)$

25. $(9p^3 + 3q^2 + 2r)(5p^2qr^3)$

26. $5xy^3 \cdot (2x^4 + 3x^2y^4 - 9x^2y)$

27. $2ab(4a^2b^3 + 6ab^2) + 4a^2b^3(5ab+7)$

28. $6y^2z^3(3y^2z^2 + 8y^2z^3) + 3y^4z(2z^5 + 3z^4)$

29. $3x^4y^5(5x^2y^3 - 2xy^2) + 2x^2y^3(x^3y^4 - 8x^4y^5)$

30. $(-2a^4b^3)(5ab - 3ab^3 + 4a^2b)$

31. $(8x^3y^5 - 4x^2y + 6x^3y^2)(-3x^2y)$

32. $5ab(6a^2b^3 + 4ab^2 + 6a^3b + 4a^3b^4)$

33. $(6x^8y^7 - 3x^5y^6 - 5x^4y^5 + 8)(-4x^2y^3)$

34. $-2x^{-3}y^2(5x^4y^{-1} - 3x^{-2}y^{-2} + 4x^3y^{-3})$

35. $5a^{-3}b^{-4}(6ab - 4a^3b^5 - 3a^4b^4 + 2a^6b^7)$

The answers to the previous exercises may be found on pages 151 and 152.

Division by Monomials

In elementary algebra you will learn applications of the distributive law which are more general than those given in the preceding sections. In this part we will learn how to divide polynomials by monomials. The principle involved could be called a distributive law for division or it can also be viewed as the process of adding fractions in reverse.

To add or subtract numeric fractions first requires that the fractions be written with a common denominator. The same is true of algebraic fractions. As long as we have common denominators we simply add or subtract the numerators. Consider the following simple numerical example.

$$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7}$$

Reading this example from right to left shows how to distribute the 7 under both the 3 and the 2.

Next, consider the case where we have a numerator consisting of a sum of three terms over a single denominator.

$$\frac{4 + 3 + 6}{17} = \frac{4}{17} + \frac{3}{17} + \frac{6}{17}$$

This example shows how to distribute the single denominator under each of the three numerators.

The same process applies when the numbers are replaced by algebraic terms or expressions. Such fractions are called algebraic fractions and we will now demonstrate how to simplify certain algebraic fractions.

As in multiplying monomials, when we divide monomials we should be concerned with the following order:

SIGNS NUMBERS VARIABLES IN ALPHABETIC ORDER

When dividing one power of a variable by another power of the same variable we must remember to subtract exponents. We should subtract the bottom exponent from the top exponent.

Examples: $\frac{15x^5}{3x^2} = 5x^{5-2} = 5x^3$

$$\frac{-42a^3b^4c^5}{7a^2bc^3} = -6a^{3-2}b^{4-1}c^{5-3} = -6ab^3c^2$$

$$\frac{24p^6q^3r^4}{-6p^2qr^2} = -4p^{6-2}q^{3-1}r^{4-2} = -4p^4q^2r^2$$

The same process applies when the exponents are negative numbers. Particular care must be exercised when the bottom exponent is negative, for the net effect of subtracting a negative number is to add a positive number. Also, if the top and bottom exponents are the same then when we subtract we get 0 for an exponent. But $x^0 = 1$, or any variable with a zero exponent equals one. The net effect is that the variable disappears. Study the following examples.

Examples: $$\frac{14x^4y^{-3}}{2x^{-2}y^{-5}} = 7x^{4-^-2}y^{-3-^-5}$$

$$= 7x^{4+^+2}y^{-3+^+5} = 7x^6y^2$$

$$\frac{-18a^3b^2c^5}{-6a^{-1}b^2c^{-2}} = 6a^{3-^-1}b^{2-2}c^{5-^-2}$$

$$= 6a^{3+^+1}b^0c^{5+^+2}$$

$$= 6a^4c^7$$

$$\frac{-12x^4y^2z^{-3}}{-3x^2y^{-2}z^{-3}} = 4x^{4-2}y^{2-^-2}z^{-3-^-3}$$

$$= 4x^2y^{2+^+2}z^{-3+^+3}$$

$$= 4x^2y^4z^0 = 4x^2y^4$$

To divide a polynomial by a monomial we first apply the process illustrated by the numerical fraction examples at the beginning of this section. Then we divide each pair of monomials as illustrated in the previous examples.

Examples: $$\frac{12x^6 + 18x^4 + 16x^3}{2x^2}$$

$$= \frac{12x^6}{2x^2} + \frac{18x^4}{2x^2} + \frac{16x^3}{2x^2}$$

$$= 6x^4 + 9x^2 + 8x$$

$$\frac{9a^3b^2 + 12a^5b^4}{3a^2b^{-3}}$$

$$= \frac{9a^3b^2}{3a^2b^{-3}} + \frac{12a^5b^4}{3a^2b^{-3}}$$

$$= 3a^{3-2}b^{2-^-3} + 4a^{5-2}b^{4-^-3}$$

$$= 3ab^5 + 4a^3b^7$$

EXERCISES: Perform the following divisions. Where possible add or subtract like terms.

1. $\frac{6x^3}{2x}$

2. $\frac{15a^6}{3a^{-2}}$

3. $\frac{20a^3b^2c}{4a^2bc^3}$

4. $\frac{26x^5y^7z^4}{-2x^3y^2z^{-1}}$

5. $\frac{16x^6y^2}{2x^2y^{-1}} + \frac{12x^5y^6}{4xy^3}$

6. $\frac{-30a^6b^3c^7}{6a^2bc^{-1}} - \frac{38a^5b^8c^5}{19ab^6c^{-3}}$

7. $\frac{56w^4z^7}{-28w^2z^2} + \frac{-60w^5z^6}{12w^3z} - \frac{44w^6z^{-2}}{11w^4z^{-7}}$

8. $\frac{34a^5b^6}{17a^2b^3} - \frac{-39a^{-1}b^{-4}}{13a^{-4}b^{-1}} + \frac{10a\,b^{-1}}{5a^{-2}b^{-4}}$

9. $\frac{25x^2z^7}{5z^2} + \frac{35x^5z^{-2}}{7x^3z^{-3}} - \frac{21x\cdot z^3}{3x^{-1}z^{-2}} - \frac{28x^3z^5}{14x\cdot z^4}$

10. $\dfrac{46x^5 + 36x^3}{2x^2}$

11. $\dfrac{40a^6b^4 + 24a^7b^2}{8a^3b^{-1}}$

12. $\dfrac{12x^2y^3z^5 + 24x^5y^6z^2 + 36x^3y^5z^3}{6x^2y^3z}$

13. $\dfrac{21a^5b^4c + 35a^3b^4c^6 + 49a^5bc^6}{7a^3bc^4}$

14. $\dfrac{9a^3b^6 + 6a^5b^7}{3a^2b^3} + \dfrac{12a^8b^7 + 20a^{10}b^8}{4a^7b^4}$

15. $\dfrac{20x^8y^5 - 8x^6y^6}{2x^3y^2} + \dfrac{27x^4y^6 - 45x^6y^5}{9xy^2}$

16. $\dfrac{32p^3r^2 - 48p^7r - 56pr^5}{-8p^5r^3}$

17. $\dfrac{20x^5z^7 + 16x^3z^5 - 32x^6z^4 - 36xz^3}{4x^2z^3}$

18. $\dfrac{36a^5b^2c^7 - 18a^3b^3c^4 + 54a^6b^5c^6 - 81a^5b^7c^9}{9a^3b^2c^4}$

The answers to the previous exercises may be found on page 152.

LITERAL FORMULAS

A literal formula is an equation which expresses a relationship between two or more variables. The purpose of this section is to introduce the use of formulas and provide practice in substituting numbers for variables in a variety of formulas.

Example: $F = \frac{9}{5} C + 32$

This equation can be used to convert degrees centigrade (C) to degrees Fahrenheit (F). By substituting $C = 80$ in the right side of the above formula we find $F = \frac{9}{5} \cdot 80 + 32 = 176$. 80° centigrade corresponds to 176° Farenheit.

Example: The above formula is meant to be used to convert from centigrade to Fahrenheit. A second formula can be used to convert from Fahrenheit to centigrade.

$$C = \frac{5}{9} (F - 32)$$

To find the equivalent of 5° Fahrenheit we substitute $F = 5$ and then compute the resulting value of C.

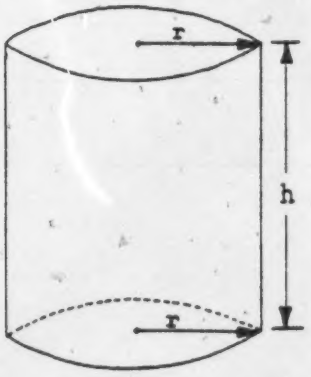
$C = \frac{5}{9} (5 - 32) = \frac{5}{9} (-27) = -15$

5° Fahrenheit corresponds to -15° centigrade.

In algebra you will learn how to derive and manipulate formulas such as those above. It is usually possible to derive a separate formula for each variable that occurs in a literal formula.

Example: $V = \pi r^2 h$

This formula gives the volume of a cylinder in terms of its radius (r) and height (h). π is a constant which may be assumed to have the value $\pi = 3.142$

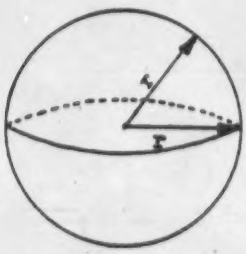


If $r = 7$ and $h = 10$ then

$$V = \pi r^2 h$$
$$= (3.142)(7^2)(10)$$
$$= 1629.580 \text{ cubic inches}$$

Example: $V = \frac{4}{3}\pi r^3$

This formula gives the volume of a sphere in terms of its radius (r). Again we assume $\pi = 3.142$.



If the radius of a basketball is 4.5 inches, find its volume.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.5)^3$$

$$\approx 381.704 \text{ cubic inches.}$$

Example: $A = P(1 + i)^n$

If you invest (P) dollars at an interest rate (i), then compounded annually the total amount (A) at the end of (n) years is given by the above formula. The interest rate i should be expressed as a decimal.

If you put \$400 in a savings account which earns 6% interest, then the total amount after 3 years is given by:

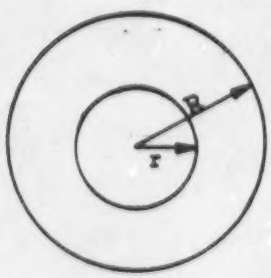
$$A = P(1 + i)^n = 400\ (1 + .06)^3$$

$$= 400\ (1.191016)$$

$$\approx \$476.4$$

Example: $A = \pi R^2 - \pi r^2$

This formula gives the area of the ring between two concentric circles.



Find the ring area if $R = 8$ and $r = 5$. Assume $\pi = 3.142$.

$$A = \pi R^2 - \pi r^2 = \pi \cdot 8^2 - \pi 5^2$$

$$= (3.142)(64) - (3.142)(25)$$

$$= 201.088 - 78.55$$

$$= 122.538$$

Note the distinction in this example of the upper case and lower case letters R and r. In algebra it is important to make a distinction between upper and lower case letters. Never use an upper case letter if a lower case letter is given.

Example: Young's Rule is a formula which can be used to determine the drug dosage for a child, given the age of the child and the standard dosage for an adult.

$$C = \frac{AD}{A+12}$$

where C = dosage for the child
D = dosage for an adult
A = age of the child

If the usual dosage for an adult is 8 grams find the dosage for a 6 year old.

$$C = \frac{6 \cdot 8}{6+12} = \frac{48}{18} = 2\frac{2}{3} \text{ grams}$$

Example: $h = -16t^2 + st$

This formula gives the height (h) which an object will reach in (t) seconds if it is projected with an initial speed (s). Use the formula to find the height of a ball thrown upward with a speed of 60 feet per second at the end of 2 seconds.

$$\begin{aligned} h &= -16(2)^2 + 60(2) \\ &= -16 \cdot 4 + 120 \\ &= 56 \end{aligned}$$

The height of the ball after 2 seconds is 56 feet.

Each of the previous examples consisted of specific applications of formulas. The problems below also contain formulas, but the uses of these formulas will not be explained here. Their only purpose is to provide practice in substituting numbers for letters and provide practice in using the order of operations implied by formulas.

Example: Given $x = -2$, $y = 6$ and $z = -8$, compute

$$V = x^2 - y^2 + (x-y)^2 + z^2$$

We make uniform substitutions for each letter. When substituting a negative number for a letter you will find it convenient to enclose the number and its sign within parentheses.

$$
\begin{aligned}
V &= (^{-}2)^2 - 6^2 + (^{-}6-6)^2 + (^{-}8)^2 \\
&= \quad 4 \quad - 36 + (^{-}12)^2 \quad + 64 \\
&= \qquad - 32 + 144 \quad + \quad 64 \\
&= \qquad\quad 112 + 64 \\
&= \qquad\quad 176
\end{aligned}
$$

It is probably best to first substitute for each letter and write down the first line in a form as close to the original formula as possible. Avoid trying to do computations at the same time when substitutions are made. Then write out a sequence of steps which shows the operations being carried out. Don't be in a hurry to get the final answer. At this stage it is probably best to do only one or two operations before writing down a new line.

Example: Find T if, $T = (((5x + 3)\ x + 6)\ x - 4)\ x + 8$,
where $x = {}^{-}3$.

This example already contains grouping symbols and it is important to include these when writing down the first line of the substitution.

$$
\begin{aligned}
T &= (((5(^{-}3) + 3)(^{-}3) + 6)(^{-}3) - 4)(^{-}3) + 8 \\
&= (((^{-}15 + 3)(^{-}3) + 6)(^{-}3) - 4)(^{-}3) + 8 \\
&= (((^{-}12)(^{-}3) + 6)(^{-}3) - 4)(^{-}3) + 8 \\
&= ((36 + 6)(^{-}3) - 4)(^{-}3) + 8 \\
&= (42(^{-}3) - 4)(^{-}3) + 8 \\
&= (^{-}126 - 4)(^{-}3) + 8 \\
&= (^{-}130)(^{-}3) + 8 \\
&= 390 + 8 \\
&= 398
\end{aligned}
$$

Example: If $a = 2$ and $b = 3$, compute $F = \dfrac{a^2 + ab}{1 + a+b}$

$$F = \frac{(^-2)^2 + (^-2)\cdot 3}{1 + (^-2)+3} = \frac{4 + ^-6}{1 + -\frac{2}{3}}$$

$$= \frac{^-2}{\frac{1}{3}}$$

$$= ^-2\cdot(\tfrac{3}{1})$$

$$= ^-6$$

EXERCISES: The first eight problems refer to the first eight examples in this section.

1. The melting point of aluminum is 660°C. What is this temperature in degrees Fahrenheit?

2. The normal body termperature is 98.6°F. What is this temperature in degrees centigrade?

3. Find the volume of a can of noodles if the can is 9 inches tall and has a circular radius of 2 inches.

4. Find the volume of a sphere if its radius is 12 inches.

5. Assume you earn 8% interest compounded annually on a special savings account. If you deposit $800, what should be the total in the account at the end of 2 years.

6. Find the area of a ring if its outer radius is 18 inches and its inner radius is 9 inches.

7. The standard dosage for an adult of a certain drug is 6 grams. Find the dosage for a 4 year old.

8. If baseball is thrown in the air with an initial speed of 54 feet per second, how high is the ball at the end of 1.5 seconds?

Find the value of the indicated letter in problems 9-30. Use the values: $x = ^-2$, $y = 5$, $z = ^-6$, $a = 4$, $b = ^-10$, $c = ^-3$.

9. $w = \dfrac{z}{x+y}$

10. $w = \dfrac{x^2+y^2}{x^2-y^2}$

11. $d = \dfrac{a^2-b^2}{a^2+b^2}$

12. $w = \dfrac{x^3-y^3}{x^2-y^2}$

13. $d = \dfrac{\frac{a}{a+b} - a}{b - \frac{a}{a+b}}$

14. $v = \dfrac{x + \frac{x}{y}}{x^2 - \frac{x^2}{y^2}}$

15. $s = \frac{x^2 + xy}{x^2 - xy}$

16. $d = \frac{1}{a} + \frac{1}{b}$

17. $w = x(y+x^2)$

18. $d = \frac{a}{b(a-b)}$

19. $d = \frac{ab}{a+b} + \frac{a+b}{ab}$

20. $w = 3x^2 + 5x - 15$

21. $d = \sqrt{b^2 - 4ac}$

22. $w = 6x(y+z)^3$

23. $d = \frac{ab}{\sqrt{a^2+b^2}}$

24. $m = \frac{6x - b(a+c)^2}{10y - z(y+z)}$

25. $t = -2[x + 2(x+y)^2]$

26. $p = (((3x+5)x+4)x-2)x+6$

27. $p = ((((6z+2)z-1)z+4)z+2)z-3$

28. $r = 6(a+6)^2 - c(a+b^2)$

29. $v = \frac{ax + by^2 + cz^3}{2bc + 4a^2}$

30. $v = \frac{ax^2 + bx + c}{(a(x+by)+c)z}$

The answers to the previous exercises may be found on pages 152 and 153.

SECTION VI REVIEW PROBLEMS

Combine like terms in problems 1-10.

1. $19x + 12x$
2. $-6w^2 + 4w^2$
3. $17m - m + 5m$
4. $a^3 + 5a^3$
5. $14x^2 - 7x + 13 - 9x^2 + 15x - 2$
6. $9ab^2 + 12a^2b + 8ab^2 + 13a^2b$
7. $17x^3 - (4x + 2 - 5x^3) + 9x + 4x^2$
8. $9y^2 + 3y - 7 - (8 - 5y - 2y^2)$
9. $5a^3b - 3ab^3 - (11a^3b + 3ab^3) - (6ab^3 - 2a^3b)$
10. $7x^4 + 5x^3 - 2x^2 + 3x - (5x - 9x^3 + 4x^2 - 3x^4)$

Multiply the terms in problems 11-25. If necessary use the Distributive Law and then combine like terms.

11. $(4a^3)(2a^2)$
12. $(3x^2)(-5x^4)$
13. $(-8y^3)(-3y^5)$
14. $(5a^2b)(-3ab^3)$
15. $(-2x^2y^3)(5x^3y^5)$
16. $(-8w^2z^4)(-2w^6z^2)$
17. $(4a^3)(-3a^4) + (7a^2)(4a^5) + (-6a)(2a^6)$
18. $13x^3 \cdot 3x^2 - 5x \cdot 6x^4 + 8x^2 \cdot 4x^3$
19. $(4a^2b^4)(2a^3b^3) + (11ab^2)(-2a^4b^5) + (-3a^3b)(-5a^2b^6)$
20. $5x^2(3x^3 - 2x^2 + 7x - 4)$
21. $8a^2b^3(2ab + 3a^3b - 5ab^2)$
22. $(2z^7 - 5z^5 + 3z^3 + 2z)(4z^3)$
23. $4y^3(3y^2 - 5y^3 - 6y^4)$
24. $7z^5(2 + 3z^2 - 5z^3 + 2z^4)$
25. $3x^3y^4(5x^2y^3 - 6x^3y^5 + 2xy^2 - 8x^5y^3)$

Perform division in problems 26-35. Where possible, add or subtract like terms.

26. $\frac{9x^5}{3x^2}$

27. $\frac{18z^5}{2z^{-2}}$

28. $\frac{36a^5b^4c^7}{9a^2b^{-1}c^{-3}}$

29. $\frac{44x^3y^2z^5}{11x^{-1}y^2z^4}$

30. $\frac{68a^{-1}b^{-2}c^{-3}}{17^2\ b^{-5}c^{-6}}$

31. $\frac{16x^2y^3}{4x^{-6}y^{-1}} + \frac{28x^5y^6}{2x^{-3}y^2}$

32. $\frac{30a^2b^3c^{-1}}{6a\ b^3c^{-4}} + \frac{48a^{-1}b^5c^6}{8a^{-2}b^5c^3}$

33. $\frac{18x^7 + 20x^5 + 24x^4}{2x^2}$

34. $\frac{58a^3 - 42a^6 - 24a^8}{-2a^{-1}}$

35. $\frac{27x^3y^4 - 15x^7y^6 + 18xy^5 - 33x^4y^2}{-3x^{-1}y^2}$

36. Compute: $d = vt + \frac{1}{2}at^2$ given: $v = 12,\ t = 3,\ a = 12.$

37. Compute: $T = 2\pi\sqrt{\frac{I}{wr}}$ given: $\pi = 3.142,\ I = 216$

$w = 2,\quad r = 3$

38. Compute: $E = \frac{1}{2}mv^2$ given: $m = 200,\ v = 28$

39. Compute: $A = 2\pi rh + \pi r^2$ given: $\pi = 3.142,\ r = 4,\ h = 10.$

40. Compute: $D = \sqrt{b^2 - 4ac}$ given: $b = 12,\ a = -6,\ c = 8.$

41. Compute: $V = \frac{1}{3}\pi r^2h$ given: $\pi = 3.142,\ r = 6,\ h = 15.$

42. Compute: $x = \frac{1}{2\pi fc}$ given: $\pi = 3.142,\ f = 4800,\ c = 6.$

43. Compute: $e = \frac{Pl}{AE}$ given: $P = 50,\ l = 18,\ A = 4,$

$E = 6000$

44. Compute: $R = \left[\frac{2x^2 + 3x + 5}{x^2 + 2x + 1}\right]^2$ given: $x = -3$

45. Compute: $p = ((((5x-4)x+2)x-3)x+6)x-5$ given: $x = -4$

The answers to the Review Problems may be found on page 154.

SECTION I ANSWERS

From page 1

1. 350
2. 521
3. 141,846
4. 4,024
5. 37,575
6. 17,453,042
7. 68,629
8. 70,430
9. 21,072
10. 168,015
11. 15,015
12. 1,021,108
13. 3,742,782
14. 4,530,008
15. 156 times a year
16. 1,095 times a year
17. 985 calories
18. 3,842,900 meters
19. 64,418 gallons
20. 31,680 gallons

From page 3

1. 8 R29
2. 35 R5
3. 39 R10
4. 5 R1
5. 14 R3
6. 26 R6
7. 285 R14
8. 138 R15
9. 16 R49
10. 33 R93
11. 34 R3
12. 9 R402
13. 68 R240
14. 62 R172
15. 58 R556
16. 4 R485
17. 5002
18. 90 R880
19. 8004 R4
20. 8 R244

From page 6

1.	2 no	3 no	5 yes	10 no	7.	2 no	3 yes	5 no	10 no
2.	2 yes	3 yes	5 yes	10 yes	8.	2 no	3 no	5 no	10 no
3.	2 yes	3 no	5 yes	10 yes	9.	2 no	3 no	5 yes	10 no
4.	2 yes	3 yes	5 no	10 no	10.	2 no	3 no	5 yes	10 no
5.	2 yes	3 yes	5 yes	10 yes	11.	2 yes	3 no	5 no	10 no
6.	2 yes	3 no	5 yes	10 yes	12.	2 no	3 no	5 no	10 no

From page 6

13. 2 yes 3 yes 5 yes 10 yes
14. 2 yes 3 no 5 no 10 no
15. 2 no 3 no 5 no 10 no
16. 2 no 3 no 5 yes 10 no
17. 2 no 3 no 5 yes 10 no
18. 2 yes 3 no 5 yes 10 yes
19. 2 no 3 yes 5 no 10 no
20. 2 yes 3 no 5 yes 10 yes

From page 8

1. 8
2. 9
3. 343
4. 256
5. 125
6. 64
7. 36
8. 0
9. 1
10. 0
11. 1
12. 0
13. 36
14. 81
15. 81
16. 64
17. 400
18. 1960
19. 12,375
20. 129
21. 300
22. 3,600
23. 2,646
24. 11,025
25. 4,840
26. 900
27. 729
28. 1,050
29. 22,932
30. 17,640

From page 11

1. 4
2. 33
3. 182
4. 210
5. 88
6. 34
7. 64
8. 64
9. 532
10. 16,632
11. 126
12. 21
13. 0
14. 205
15. 137
16. 13,133
17. 2,081
18. 7564
19. 62,275
20. 21
21. 12
22. 6
23. 8
24. 282
25. 8,180

From page 16

1. $2^1 \times 3^1 \times 5^1 \times 7^1$
2. $2^3 \times 3^1 \times 5^1$
3. $2^2 \times 3^2 \times 5^2$
4. $2^3 \times 5^2 \times 7^1 \times 11^1$
5. $3^2 \times 5^1 \times 7^1$
6. $5^1 \times 7^2 \times 11^1$
7. $7^1 \times 11^1 \times 13^1$
8. $2^1 \times 11^1 \times 13^1$
9. $2^3 \times 5^1 \times 11^1$
10. $2^1 \times 13^1 \times 17^1$
11. $2^2 \times 3^3$
12. $2^2 \times 5^3$
13. $3^2 \times 7^2 \times 11^1$
14. $19^1 \times 23^1$
15. $3^1 \times 31^1$
16. 53^1
17. 229^1
18. $2^1 \times 89^1$
19. $3^1 \times 53^1$
20. $2^2 \times 5^2 \times 7^3$
21. $2^1 \times 5^4 \times 11^1$
22. $5^1 \times 7^1 \times 13^2$
23. $7^2 \times 19^1 \times 23^1$
24. $17^1 \times 29^1$
25. 31^2

From page 18

1. LCM = $2^4 \times 3^3 = 432$
2. LCM = $2^2 \times 3^2 \times 7^1 = 252$
3. LCM = $2^3 \times 3^1 \times 7^1 = 168$
4. LCM = $2^6 \times 3^2 = 576$
5. LCM = $2^2 \times 3^1 \times 13^1 = 156$
6. LCM = $2^3 \times 3^1 \times 5^1 = 120$
7. LCM = $2^1 \times 3^3 \times 5^1 = 270$
8. LCM = $2^2 \times 5^1 \times 23^1 = 460$
9. LCM = $2^1 \times 13^1 \times 31^1 = 806$
10. LCM = $2^3 \times 3^2 \times 5^3 = 9000$

From page 19

1. GCD = $2^4 \times 3^1 \times 5^1 = 60$
2. GCD = $2^1 \times 3^1 = 6$
3. GCD = $3^1 \times 7^1 = 21$
4. GCD = 11
5. GCD = $2^1 \times 3^2 \times 5^2 = 450$
6. GCD = $2^4 \times 5^1 = 80$
7. GCD = $2^1 \times 13^1 = 26$
8. GCD = $3^1 \times 5^1 \times 7^1 = 105$

From page 20

9. GCD = $5^1 \times 7^1 = 35$

10. GCD = $2^2 \times 3^1 \times 7^1 = 84$

From page 24

1. 7x4 + 7x9 = 91
2. 8x6 + 8x3 = 72
3. 13x4 + 13x7 = 143
4. 8x5 + 8x6 = 88
5. 14x5 + 14x3 = 112
6. 13 x (9+3) = 156
7. 7 x (11+16) = 189
8. 6 x (9+14) = 138
9. 15 x (3+1) = 60
10. 9 x (5+15) = 180
11. 11x4 - 11x2 = 22
12. 8 x (13-11) = 16
13. 15 x (7+3) = 150
14. 19 x (3+17) = 380
15. 37 x (5+15) = 740
16. 4x5 + 4x8 + 4x11 = 96
17. 3x2 + 3x7 + 3x6 = 45
18. 13 x (3+5+7) = 195
19. 47 x (2+8+10) = 940
20. 51 x (6+3+4) = 663

SECTION I Review

From page 25

1. 22784
2. 479672
3. 192 R23
4. 53 R79
5. 788 R501
6. 2 no 3 yes 5 no 10 no
7. 2 no 3 no 5 no 10 no
8. 2 no 3 no 5 yes 10 no
9. 2 no 3 yes 5 yes 10 no
10. 2 yes 3 no 5 no 10 no
11. 2 yes 3 yes 5 no 10 no
12. 2 yes 3 no 5 yes 10 yes
13. 2 yes 3 no 5 yes 10 yes
14. 200
15. 288
16. 576
17. 2475
18. 29645
19. 5764
20. 0
21. 316
22. 231
23. 41
24. 134
25. 10541
26. 3
27. 59
28. 24
29. 14432
30. 84
31. GCD = $2^3 \times 3^1 \times 5^1 = 120$
    LCM = $2^4 \times 3^2 \times 5^2 = 3600$
32. GCD = $2^2 \times 3^1 \times 5^1 = 60$
    LCM = $2^4 \times 3^2 \times 5^2 = 3600$
33. GCD = $3^1 \times 7^2 = 147$
    LCM = $2^1 \times 3^3 \times 7^2 = 2646$
34. GCD = $5^1 \times 7^1 = 35$
    LCM= $5^2 \times 7^1 \times 11^1 \times 13^1 =$ 25025
35. GCD = $3^1 \times 7^1 = 21$
    LCM = $2^5 \times 3^1 \times 5^1 \times 7^1 \times 11^1 =$ 36960
36. GCD = $2^3 \times 17^1 = 136$
    LCM = $2^3 \times 5^1 \times 7^1 \times 17^1 =$ 4760
37. GCD = $5^1 \times 13^1 = 65$
    LCM = $3^2 \times 5^2 \times 13^1 = 2925$
38. GCD = $2^1 \times 3^1 = 6$
    LCM = $2^2 \times 3^2 \times 5^1 \times 7^2 \times 13^1 =$ 114660
39. 72
40. 140
41. 105
42. 140
43. 260
44. 150
45. 570
46. 690
47. 108
48. 240
49. 620
50. 1740

SECTION II ANSWERS

From page 32

1. 4
2. 4
3. 4
4. 17
5. 13
6. 6
7. 9
8. 12
9. 8
10. 11
11. 11
12. 5
13. 22
14. 33
15. 23
16. $3 < 5$
17. $3 > -5$
18. $-12 < -6$
19. $-1 < 10$
20. $10 > -1$
21. $-20 > -25$
22. $33 > -8$
23. $-21 > -22$
24. $5 > -6$
25. $-5 > -7$

From page 34

1. 17
2. -89
3. 17
4. -173
5. 49
6. -59
7. -228
8. 99
9. 104
10. -351
11. -175
12. -107
13. -1523
14. 359
15. -208
16. 260
17. -289
18. -1
19. 0
20. -210
21. -385
22. -507
23. 569
24. 211
25. 913

From page 37

1. -22
2. -11
3. -98
4. 16
5. 2
6. -20
7. -70
8. -122
9. -69

From page 37

10. -114
11. 960
12. -622
13. -19
14. 1594
15. -1204
16. 1263
17. -34
18. -2513
19. 33
20. 167
21. -400
22. 1230
23. 2377
24. 223
25. -83
26. 1st floor below ground
27. 20,000 feet
28. 490 feet below sea level
29. 5° below zero
30. -$24.
31. 74 units
32. 829 units

From page 39

1. 12
2. -5
3. 2
4. -5
5. -16
6. 4
7. -3
8. -52
9. 64
10. 18
11. 0
12. -64
13. -15
14. -70
15. -70
16. -24
17. -55
18. -12
19. -38
20. 7
21. 24
22. 228
23. 25
24. 49
25. 2
26. -14
27. -11
28. 44
29. 576
30. 16

SECTION II Review

From page 40

1. $-4 < 3$
2. $5 > -6$
3. $-8 < -7$
4. $-8 > -9$
5. $-25 < 24$
6. $23 > -24$
7. $-12 < 2$
8. $258 > -459$
9. $-37 < 0$
10. $26 > -43$
11. -3
12. -6
13. 10
14. -18
15. 22
16. -32
17. 294
18. -297
19. 66
20. -29
21. 30
22. 11
23. 9
24. 5
25. 8
26. 15
27. -4
28. 3
29. -51
30. -17
31. 33
32. 10
33. 80
34. -4
35. 8
36. 76
37. -22
8. -7
39. -180
40. -24
41. 2535
42. 94
43. -8
44. -141
45. -57
46. -544
47. 220
48. 840
49. 727
50. -53748

SECTION III ANSWERS

From page 45

1. $\frac{8}{9}$	7. $\frac{2}{11}$	13. $\frac{7}{9}$	19. $\frac{17}{38}$	25. $\frac{77}{126}$
2. $\frac{5}{7}$	8. $\frac{11}{35}$	14. $\frac{8}{5}$	20. $\frac{13}{44}$	26. $\frac{25}{205}$
3. $\frac{2}{5}$	9. $\frac{16}{45}$	15. $\frac{13}{11}$	21. $\frac{54}{69}$	27. $\frac{231}{168}$
4. $\frac{7}{13}$	10. $\frac{12}{29}$	16. $\frac{13}{11}$	22. $\frac{30}{78}$	28. $\frac{54}{96}$
5. $\frac{1}{3}$	11. $\frac{3}{37}$	17. $\frac{11}{18}$	23. $\frac{56}{96}$	29. $\frac{136}{384}$
6. $\frac{10}{43}$	12. $\frac{5}{21}$	18. $\frac{4}{15}$	24. $\frac{56}{32}$	30. $\frac{75}{84}$

From page 48

1. no	7. yes	13. $-2\frac{9}{13}$	19. 11	25. $-\frac{26}{3}$
2. yes	8. yes	14. $-8\frac{1}{6}$	20. -9	26. $\frac{155}{13}$
3. no	9. no	15. $2\frac{4}{7}$	21. $\frac{11}{4}$	27. $-\frac{41}{9}$
4. no	10. no	16. $-8\frac{16}{27}$	22. $\frac{40}{7}$	28. $-\frac{67}{5}$
5. no	11. $2\frac{1}{6}$	17. $-6\frac{8}{13}$	23. $\frac{55}{8}$	29. $-\frac{55}{2}$
6. no	12. $3\frac{5}{8}$	18. $6\frac{3}{19}$	24. $-\frac{31}{6}$	30. $\frac{161}{5}$

From page 50

1. 5	5. 6	9. -9	13. 18	17. 51
2. 16	6. 26	10. 4	14. 32	18. 48
3. 6	7. 25	11. -40	15. 9	19. -19
4. 4	8. -16	12. 5	16. 375	20. 216

From page 54

1. $\frac{7}{9}$	6. $3\frac{7}{15}$	11. $-1\frac{3}{4}$	16. $-\frac{3}{10}$	21. $-\frac{37}{72}$
2. $\frac{1}{2}$	7. $-5\frac{2}{15}$	12. $\frac{1}{14}$	17. $\frac{50}{63}$	22. $-4\frac{13}{60}$
3. $-\frac{1}{12}$	8. $-1\frac{7}{8}$	13. $1\frac{1}{3}$	18. $-\frac{11}{36}$	23. $-\frac{3}{26}$
4. $1\frac{8}{21}$	9. $\frac{41}{42}$	14. $-1\frac{5}{6}$	19. $\frac{41}{42}$	24. $-2\frac{19}{168}$
5. $-4\frac{7}{12}$	10. $-1\frac{5}{6}$	15. $-1\frac{29}{72}$	20. $6\frac{11}{12}$	25. $4\frac{25}{108}$

From page 56

1. $\frac{8}{63}$	6. $-\frac{9}{15}$	11. $1\frac{1}{2}$	16. $-1\frac{23}{39}$	21. $-\frac{3}{5}$
2. $1\frac{3}{5}$	7. $-12\frac{3}{4}$	12. $\frac{7}{16}$	17. $\frac{1}{5}$	22. $1\frac{1}{2}$
3. $-\frac{5}{56}$	8. $2\frac{3}{4}$	13. $1\frac{11}{17}$	18. 0	23. $\frac{2}{3}$
4. 9	9. $21\frac{3}{4}$	14. $\frac{29}{155}$	19. $2\frac{11}{20}$	24. $\frac{-43}{450}$
5. $2\frac{7}{64}$	10. $-59\frac{1}{2}$	15. $-18\frac{1}{2}$	20. $19\frac{1}{3}$	25. $1\frac{17}{38}$

From page 59

1. $\frac{8}{9}$
2. 4
3. $-1\frac{5}{21}$
4. $\frac{-14}{135}$
5. $3\frac{18}{31}$
6. $2\frac{12}{13}$
7. $\frac{1}{6}$
8. $-\frac{7}{24}$
9. $\frac{-4}{15}$
10. $\frac{-11}{15}$
11. 6
12. $1\frac{4}{39}$
13. 34
14. $-\frac{7}{108}$
15. $-\frac{8}{27}$
16. $\frac{1}{30}$
17. $3\frac{6}{19}$
18. $\frac{-184}{245}$
19. $5\frac{9}{11}$
20. $-6\frac{1}{4}$

SECTION III REVIEW

From page 61

1. $\frac{3}{7}$
2. 18
3. $-\frac{2}{7}$
4. $\frac{2}{3}$
5. $\frac{61}{304}$
6. $-\frac{7}{51}$
7. $\frac{36}{5}$
8. $\frac{2}{25}$
9. $\frac{13}{17}$
10. $\frac{5}{19}$
11. $\frac{18}{27}$
12. $\frac{40}{56}$
13. $\frac{60}{75}$
14. $\frac{84}{126}$
15. $\frac{121}{187}$
16. no
17. no
18. yes
19. no
20. no
21. 70
22. 10
23. 8
24. 120
25. -3
26. -2
27. 2
28. 63
29. 105
30. 312
31. $1\frac{5}{24}$
32. $\frac{1}{18}$
33. $\frac{9}{14}$
34. $1\frac{2}{3}$
35. $\frac{11}{36}$
36. $-1\frac{5}{24}$
37. $11\frac{11}{12}$
38. $9\frac{1}{6}$
39. $1\frac{3}{46}$
40. $\frac{9}{16}$
41. $\frac{19}{36}$
42. $\frac{1}{6}$
43. $\frac{2}{15}$
44. $7\frac{1}{9}$
45. $1\frac{2}{3}$
46. $\frac{-23}{168}$
47. $1\frac{1}{4}$
48. $1\frac{115}{216}$
49. $2\frac{59}{120}$
50. $1\frac{8}{21}$
51. $\frac{2}{3}$
52. $-2\frac{1}{2}$
53. 48
54. $-\frac{3}{16}$
55. $-\frac{1}{6}$
56. $-2\frac{8}{21}$

SECTION IV ANSWERS

From page 68

1. $\frac{43}{10}$	2. $\frac{63}{50}$	3. $-\frac{111}{50}$	4. $\frac{27}{8}$
5. $-\frac{9}{2}$	6. $\frac{47}{8}$	7. $-\frac{633}{100}$	8. $\frac{29}{8}$
9. $-\frac{213}{20}$	10. $\frac{611}{125}$	11. 17.72	12. 223.226
13. 710.237	14. 88.362	15. 16.24	16. -5.37
17. 18.058	18. -28.37	19. -3.028	20. -3.8999
21. -4.675	22. 2.7775	23. 32.4	24. -11.12
25. -0.944	26. -6.7414	27. -8.494	28. -159.82
29. -18.1896	30. 32.76		

From page 72

1. -35.511	2. -2.86	3. -10.465	4. 1979.
5. -132,610.	6. 10.92	7. 4.5796	8. -32.768
9. 81.588	10. 0.05012	11. -4.66	12. 9.568
13. -14.88	14. 153.14	15. 131.5	16. 217.672
17. 105.8841	18. 11.4668	19. -0.3025	20. 18.571
21. 0.625	22. 3.125	23. 0.325	24. 4.68
25. 3.187			

From page 76

1. 5
2. 13
3. 15
4. 26
5. 17
6. 20
7. 25
8. 34
9. $3\sqrt{2}$
10. $2\sqrt{7}$
11. $4\sqrt{2}$
12. $6\sqrt{2}$
13. $7\sqrt{2}$
14. $8\sqrt{3}$
15. $10\sqrt{5}$
16. $5\sqrt{6}$
17. $3\sqrt{13}$
18. $7\sqrt{3}$
19. $9\sqrt{3}$
20. $15\sqrt{2}$
21. $11\sqrt{3}$
22. $13\sqrt{2}$
23. $14\sqrt{2}$
24. $15\sqrt{3}$
25. $12\sqrt{3}$

From page 80

1. 0.38
2. 0.095
3. 0.0325
4. 0.0005
5. 0.055
6. 0.0675
7. 1.
8. 0.0025
9. 0.0009
10. 0.668
11. 5%
12. 200%
13. 43.2%
14. 330.%
15. 150%
16. 4%
17. 100%
18. 15%
19. 33.3%
20. 108%
21. 75%
22. 40%
23. 62.5%
24. 237.5%
25. 130%
26. 48%
27. 15%
28. 207%
29. 740%
30. 233.3%
31. $\frac{7}{20}$
32. $\frac{3}{200}$
33. $\frac{11}{8}$
34. 1
35. $\frac{5}{8}$
36. $\frac{9}{5}$
37. $\frac{3}{10}$
38. $\frac{27}{250}$
39. $\frac{5}{200}$
40. $\frac{1}{400}$

SECTION IV REVIEW

From page 82

1. $5\frac{3}{5}$
2. $\frac{12}{25}$
3. $-5\frac{473}{500}$
4. $6\frac{7}{8}$
5. $\frac{7}{250}$
6. $3\frac{33}{50}$
7. $-2\frac{7}{16}$
8. $3\frac{17}{20}$
9. $-14\frac{6}{25}$
10. $6\frac{19}{40}$
11. 5.417
12. -3.67
13. -1.37
14. 9.
15. 8.2
16. 16.8112
17. -12.
18. 29.16
19. 29.16
20. 12.167
21. -12.167
22. -4.913
23. -15.37
24. 205.
25. 0.0335
26. -20.54
27. -7.
28. -24.
29. 605.
30. -5.
31. -1.927
32. -619.2
33. 1.03
34. -9.562
35. -22.21
36. $3\sqrt{3}$
37. $4\sqrt{5}$
38. $3\sqrt{7}$
39. $6\sqrt{2}$
40. $2\sqrt{7}$
41. $10\sqrt{3}$
42. $3\sqrt{6}$
43. $4\sqrt{7}$
44. $7\sqrt{2}$
45. $8\sqrt{3}$
46. 10
47. 25
48. 25
49. 41
50. 0.875
51. 1.6875
52. 0.55
53. 1.48
54. 0.475
55. 0.42
56. 0.072
57. 0.0001
58. 0.0475
59. 0.0775
60. 8%
61. 9.3%
62. 56.7%
63. 121%
64. 0.41%
65. 50%
66. 25%
67. 60%
68. 387.5%
69. 80%
70. $\frac{11}{25}$
71. $\frac{3}{8}$
72. $\frac{3}{5}$
73. $\frac{1}{8}$
74. $\frac{9}{500}$
75. $1\frac{3}{4}$

SECTION V ANSWERS

From page 88

1. $\frac{1}{16}$
2. $\frac{1}{16}$
3. -1
4. $\frac{1}{9}$
5. $\frac{1}{125}$
6. $\frac{1}{-1000}$
7. 16
8. 64
9. $1\frac{1}{2}$
10. $-1\frac{1}{2}$
11. $-2\frac{10}{27}$
12. $\frac{4}{121}$
13. 1
14. 1
15. $5\frac{4}{9}$
16. $\frac{13}{36}$
17. $\frac{1}{25}$
18. $\frac{1}{-108}$
19. 58
20. $\frac{32}{81}$
21. 0
22. $4\frac{5}{9}$
23. $\frac{81}{256}$
24. $\frac{1}{-125}$
25. $\frac{4}{11}$
26. 11
27. -12
28. $1\frac{11}{25}$
29. $\frac{69}{104}$
30. $2\frac{12}{13}$

From page 91

1. 3^8
2. 5^2
3. 6^5
4. 6^{-1}
5. $(-2)^{-3}$
6. Undefined
7. $(-3)^{-9}$
8. $(-2)^{11}$
9. $(\frac{5}{6})^{11}$
10. $(\frac{3}{4})^{3}$
11. $(3\frac{1}{2})^{-2}$
12. 1
13. 1
14. 1
15. 2^{-3}
16. $\frac{1}{16}$
17. $\frac{1}{49}$
18. $\frac{2}{3}$
19. -2
20. $\frac{4}{25}$
21. $\frac{1}{81}$
22. 4
23. $\frac{1}{36}$
24. $\frac{1}{25}$
25. $\frac{1}{8}$
26. $\frac{1}{9}$
27. $\frac{1}{16}$
28. $-\frac{1}{27}$
29. 729
30. -128

From page 94

1. 3^{10}
2. $(-2)^6$
3. $\frac{1}{4^5}$
4. $\frac{1}{7^8}$
5. 3^8
6. $(-3)^{20}$
7. $\frac{1}{(-2)^6}$
8. $\frac{1}{3^{20}}$
9. 2^1
10. $\frac{1}{3^9}$
11. $\frac{1}{3^6}$
12. 2^{16}
13. $(\frac{3}{2})^6$
14. $(3\frac{1}{2})^6$
15. $(\frac{4}{3})^2$
16. 2^{25}
17. $(\frac{3}{8})^0$
18. $(-2\frac{2}{3})^0$
19. 4^4
20. 6^4
21. $\frac{1}{3^6}$
22. $\frac{1}{(-2)^{32}}$
23. 2^0
24. 3^2
25. $(-2)^8$
26. 0
27. $\frac{1}{(-2)^{12}}$
28. $\frac{1}{(-3)^{11}}$
29. $\frac{1}{2^8}$
30. $\frac{1}{6^{18}}$

From page 97

1. 10000	2. 0.001	3. 10000000	4. 0.0000001
5. 0.1	6. -0.01	7. 1.	8. -100000
9. 0.000001	10. 0.0001	11. 1.236	12. 15842.
13. 13.68	14. 0.000219	15. 4.37	16. 0.0000024
17. -18693000.	18. -0.00147	19. 1920000.	20. 0.000016
21. 10^{-3}	22. 10^4	23. 10^{-2}	24. 10^5
25. 10^1	26. 10^{-4}	27. 10^{-4}	28. 10^{-1}
29. 10^{-7}	30. 10^3	31. 163.631	32. -18.334
33. 16429.	34. 1215.38	35. 8010.	36. 6.3628
37. 7195.9161	38. 2609.55	39. 0.0111	40. 7424.524

From page 101

1. 2.78×10^{2}
2. -5.45321×10^{5}
3. 5.726×10^{0}
4. -5.8×10^{-1}
5. 4.89978×10^{3}
6. -8.06×10^{2}
7. -4.579×10^{3}
8. -1.64×10^{-1}
9. 9.36×10^{5}
10. 1.0858×10^{-1}
11. 1.71×10^{9}
12. 1.428×10^{2}
13. $2. \times 10^{3}$
14. -8.903×10^{-6}
15. 1.06×10^{2}
16. 4.0488×10^{-7}
17. 1.14×10^{9}
18. 1.4×10^{9}
19. -4.8×10^{11}
20. $3. \times 10^{-4}$
21. $2. \times 10^{0}$
22. 3.06×10^{-6}
23. 1.6×10^{-2}
24. 1.44×10^{-2}
25. 1.8×10^{-18}

SECTION V REVIEW PROBLEMS

From page 103

1. $\frac{1}{81}$
2. $\frac{1}{25}$
3. $\frac{1}{-8}$
4. $1\frac{7}{9}$
5. $\frac{8}{125}$
6. $\frac{1}{9}$
7. undefined
8. $1\frac{24}{25}$
9. $-4\frac{17}{27}$
10. -1
11. $-\frac{1}{72}$
12. 1
13. $1\frac{1}{3}$
14. $\frac{4}{27}$
15. $\frac{1}{216}$
16. 7
17. $\frac{18}{25}$
18. $-\frac{7}{51}$
19. 0
20. $1\frac{11}{25}$
21. 4^5
22. 5^{-1}
23. $(-2)^{-2}$
24. $(-3)^2$
25. $(\frac{2}{3})^9$
26. 4^5
27. $(-5)^0$
28. $(-2)^7$
29. 2^{14}
30. 6^{-2}
31. 2^9
32. $(-3)^{15}$
33. $(-2)^{13}$
34. 5^{-11}
35. 3^4
36. 2^{12}
37. $\frac{1}{5^6}$
38. $\frac{1}{3^{20}}$
39. 4^5
40. $\frac{1}{6^{14}}$
41. $(\frac{5}{3})^8$
42. $\frac{1}{2^{21}}$
43. 7^6
44. $\frac{1}{3^6}$

From page 103

45. 2^0

46. 1000

47. 0.01

48. 10000000

49. 10000

50. 0.001

51. 0.0001

52. 100000

53. 0.00001

54. 0.1

55. 0.000001

56. 10^4

57. 10^6

58. 10^{-2}

59. 10^{-3}

60. 10^{-1}

61. 10^3

62. 10^{-5}

63. 10^{-7}

64. 10^{-3}

65. 4.465×10^3

66. -3.9856×10^1

67. -5.319×10^5

68. 1.305×10^6

69. -1.484×10^{-1}

70. 4.93×10^7

71. 1.482×10^{-7}

72. 7.3×10^5

73. 2.4×10^{14}

74. $7. \times 10^4$

75. $4. \times 10^4$

SECTION VI ANSWERS

From page 111

1. $21x$ 2. $3a$ 3. $14y$ 4. $28z$
5. $-18x^2$ 6. $6a^2b$ 7. $22m^2$ 8. 0
9. $6w^4$ 10. $33x$ 11. $9ab^2$ 12. $-9x^2$
13. $-8a^3b$ 14. $12x + 10y$ 15. $-5m^2 + 2$ 16. $28d^2 - 31$
17. $-18x^2 + 14y^2$ 18. $-4g + 20h$ 19. $-7x^2 + 11x$
20. $-11y^2 + 8y$ 21. $-17xy$ 22. $-32x + 7y + 3xy$
23. $4a^2b + 2ab^2$ 24. $13x^3 + 6x^2 - 29x$ 25. $-10x^5$
26. $13y^3 + 14y^2 - 13y$ 27. $21zw^2$ 28. $5xy$
29. $36w^6$ 30. $40x^3y^2$ 31. $-5x^2 + 7x - 16$
32. $-3a^6 + 5a^5 + 9a^4 - 2a^3 - 3a^2 + 28a$ 33. $-41x^2y - 5$
34. $-12x^3y^4 - 24x^4y^3$ 35. $15w^3 - 7w^2 + 6w - 9$

From page 115

1. $18x^5$ 2. $-8a^4$ 3. $20x^9$ 4. $-21a^4b^3$
5. $18x^4y^6$ 6. $-42x^4y^3z^5$ 7. $24x^6y^5$ 8. $-32a^3b^4$
9. $26w^6$ 10. $4x^7$ 11. $-51a^7$ 12. 0
13. $-42w^9$ 14. $16z^6$ 15. $39x^5y^5$ 16. $-6a^5b^4+26a^3b^6$
17. $33y^5z^6 - 44y^5z^5$ 18. $38x^7y^6 - 9x^5y^2$
19. $36a^7b^3 - 39a^7b^5$ 20. $28w^6 + 20w^5 + 8w^4$
21. $27a^5 + 36a^4 - 18a^3$ 22. $24x^4 - 18x^3 - 15x^2$
23. $x^4y^6 + x^5y^5$ 24. $8a^4b^5 - 36a^5b^5 + 12a^6b^6$
25. $45p^5qr^3 + 15p^2q^3r^3 + 10p^2qr^4$ 26. $10x^5y^3 + 15x^3y^7 - 45x^3y^4$

27. $28a^3b^4 + 40a^2b^3$

28. $27y^4z^5 + 54y^4z^6$

29. $-x^6y^8 - 4x^5y^7$

30. $-10a^5b^4 + 6a^5b^6 - 8a^6b^4$

31. $-24x^5y^6 + 12x^4y^2 - 18x^5y^3$

32. $30a^3b^4+20a^2b^3+30a^4b^2+20a^4b^5$

33. $-24x^{10}y^{10} + 12x^7y^9 +20x^6y^8 - 32x^2y^3$

34. $-10xy + 6x^{-5} - 8y^{-1}$

35. $30a^{-2}b^{-3} - 20b - 15a + 10a^3b^3$

From page 119

1. $3x^2$
2. $5a^4$
3. $5abc^{-2}$
4. $-13x^2y^5z^5$
5. $11x^4y^3$
6. $-7a^4b^2c^8$
7. $-11w^2z^5$
8. $4a^3b^3 + 3a^3b^{-3}$
9. $3x^2z - 2x^2z^5$
10. $23x^3 + 18x$
11. $5a^3b^5 + 3a^4b^3$
12. $2z^4 + 4x^3y^3z + 6xy^2z^2$
13. $3a^2b^3c^{-3} + 5b^3c^2 + 7a^2c^2$
14. $6ab^3 + 7a^3b^4$
15. $5x^5y^3 - x^3y^4$
16. $-4p^{-2}r^{-1} + 6p^2r^{-2} + 7p^{-4}r^2$
17. $5x^3z^4 + 4xz^2 - 8x^4z - 9x^{-1}$
18. $4a^2c^3 - 2b + 6a^3b^3c^2 - 9a^2b^5c^5$

From page 125

1. $1220^\circ F$
2. $37^\circ C$
3. 113.112 cubic inches
4. 7239.168 cubic inches
5. $933.12
6. 763.506 square inches
7. 45 feet
8. 1½ grams
9. -2
10. $\frac{-29}{21}$
11. $\frac{-21}{29}$
12. $\frac{19}{3}$
13. $\frac{-1}{2}$
14. $\frac{-5}{8}$
15. $\frac{-3}{7}$
16. $\frac{3}{20}$
17. -18
18. $1/35$
19. $\frac{409}{60}$
20. -13
21. $2\sqrt{37}$
22. 12
23. $\frac{-20}{\sqrt{29}}$
24. $\frac{-1}{22}$

25. -32 26. 34 27. $-43,719$ 28. 528

29. $\frac{195}{62}$ 30. $\frac{11}{422}$

SECTION VI REVIEW

From page 127

1. $31x$ 2. $-2w^2$ 3. $21m$ 4. $6a^3$

5. $5x^2 + 8x + 11$ 6. $17ab^2 + 25a^2b$ 7. $22x^3 + 4x^2 + 5x - 2$

8. $11y^2 + 8y - 15$ 9. $4a^3b - 12ab^3$

10. $10x^4 + 14x^3 - 6x^2 - 2x$ 11. $8a^5$ 12. $-15x^6$

13. $24y^8$ 14. $-15a^3b^4$ 15. $-10x^5y^8$ 16. $16w^8z^6$

17. $4a^7$ 18. $15x^5$ 19. a^5b^7

20. $15x^5 - 10x^4 + 35x^3 - 20x^2$ 21. $16a^3b^4 + 24a^5b^4 - 40a^3b^5$

22. $8z^{10} - 20z^8 + 12z^6 + 8z^4$ 23. $12y^5 - 20y^6 - 24y^7$

24. $14z^9 - 35z^8 + 21z^7 + 14z^5$ 25. $15x^5y^7 - 18x^6y^9 + 6x^4y^6 - 24x^8y^7$

26. $3x^3$ 27. $9z^7$ 28. $4a^3b^5c^{10}$ 29. $4x^4z$

30. $4a^{-3}b^3c^3$ 31. $18x^8y^4$ 32. $11ac^3$ 33. $9x^5+10x^3+12x^2$

34. $-29a^4 + 21a^7 + 12a^9$ 35. $-9x^4y^2 + 15x^8y^4 - 6x^2y^3 + 11x^5$

36. 90 37. 37.704 38. 78,400 39. 301.632

40. $4\sqrt{21}$ 41. 565.56 42. 5.525×10^{-6} 43. $\frac{3}{80}$

44. $\frac{49}{4}$ 45. -6349

PRETEST ANSWERS

1. 822,432
2. 2,474,360
3. 570,528
4. 168R20
5. 609R12
6. 790R50
7. LCD = 30
8. LCD = 60
9. LCD = 72
10. 8 yards 1 foot 5 inches
11. 4 hours 35 minutes 35 seconds
12. $1\frac{5}{24}$
13. $10\frac{13}{18}$
14. $9\frac{3}{8}$
15. $1\frac{2}{15}$
16. $\frac{1}{2}$
17. $9\frac{5}{48}$
18. $4\frac{4}{9}$
19. $1\frac{1}{5}$
20. $\frac{3}{4}$
21. $2\frac{4}{5}$
22. $\frac{3}{4}$
23. 18
24. $\frac{3}{4}$
25. 7.96
26. $4\frac{17}{20}$
27. 26.796
28. 8.34
29. 50.5%
30. 0.075
31. 60%
32. 52.5
33. 40%
34. 70
35. 15
36. 24 workers
37. 16 pounds of tin
    8 pounds of lead
38. 53.75 miles per hour
39. area = 616
40. c = 10

PRE - TEST

Perform the following computations.

1. $\begin{array}{r} 8567 \\ \times 96 \\ \hline \end{array}$

2. $\begin{array}{r} 6080 \\ \times 407 \\ \hline \end{array}$

3. $\begin{array}{r} 6792 \\ \times\ 84 \\ \hline \end{array}$

4. $56\overline{)9428}$

5. $34\overline{)20718}$

6. $66\overline{)52190}$

Find the least common denominator for each pair of fractions in problems 7-9.

7. $\frac{5}{6}$ and $\frac{7}{10}$

8. $\frac{3}{10}$ and $\frac{11}{12}$

9. $\frac{13}{18}$ and $\frac{5}{24}$

Add the following measurements and write your final answer in terms of yards, feet, and inches. 1 yard = 3 feet. 1 foot = 12 inches.

10. $\begin{array}{r} 4 \text{ yards } 2 \text{ feet } 7 \text{ inches} \\ +\ 3 \text{ yards } 1 \text{ foot } 10 \text{ inches} \\ \hline \end{array}$

Subtract the following times and write your final answer in terms of hours, minutes, and seconds.

11. $\begin{array}{r} 8 \text{ hours } 12 \text{ minutes } 20 \text{ seconds} \\ -\ 3 \text{ hours } 36 \text{ minutes } 45 \text{ seconds} \\ \hline \end{array}$

Perform the indicated operations on the fractions in problems 12-24. Write the answers as reduced mixed numbers.

12. $\frac{3}{8} + \frac{5}{6}$

13. $7\frac{1}{6} + 3\frac{5}{9}$

14. $12 - 2\frac{5}{8}$

15. $2\frac{5}{6} - 1\frac{7}{10}$

16. $\frac{2}{3} \times \frac{9}{12}$

17. $2\frac{3}{8} \times 3\frac{5}{6}$

18. $8 \times \frac{5}{9}$

19. $\frac{2}{3} \times 1\frac{4}{5}$

20. $\frac{2}{3} \div \frac{8}{9}$

21. $2\frac{1}{3} \div \frac{5}{6}$

22. $3\frac{3}{8} \div \frac{9}{2}$

23. $6 \div \frac{1}{3}$

24. $3\frac{3}{4} \div \frac{5}{1}$

25. Round: 7.9582 to the nearest hundredth.

26. Write: 4.85 as a reduced mixed number.

27. $\begin{array}{r} 4.62 \\ \times 5.8 \\ \hline \end{array}$

28. $4.5\overline{)37.53}$

29. Write: 0.505 as a percent.

30. Convert: 7.5% to decimal form.

31. Write $\frac{3}{5}$ as a percent.

32. Find 62.5% of 84.

33. What percent of 125 is 50?

34. 42 is 60% of what number?

35. Find the unknown number in the proportion.

$$\frac{65}{104} = \frac{?}{24}$$

36. If 16 workers take 21 days to complete a job, how many workers would be required to complete the same job in 14 days?

37. Soft solder is composed of two parts of tin and one part of lead. How much tin and how much lead would be required to make 24 pounds of solder?

38. A car was driven 60 miles per hour for 3 hours and then 50 miles per hour for 5 hours. What was the average speed?

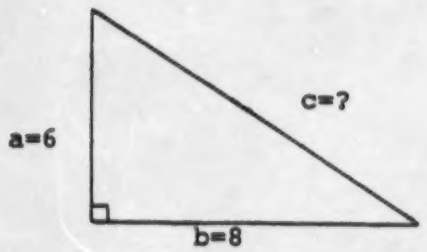
39. The formula which gives the area of a circle is:

$$A = \pi r^2$$

where r = radius of the circle. Find the area of a circle of radius $r = 14$. You should assume $\pi = \frac{22}{7}$.

40. The sides of a right triangle as pictured below are related by the the equation:

$$a^2 + b^2 = c^2$$



Find the length of side c if $a = 6$ and $b = 8$.

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